ABSTRACT

Title of Dissertation: DEVELOPMENT OF RISK-BASED MEASUREMENTS AND METRICS FOR SUSTAINABILITY QUANTIFICATION OF MANUFACTURED AND CONSTRUCTED SYSTEMS

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Dissertation directed by: Dr. Bilal Ayyub, Professor of Civil and Environmental Engineering

Sustainability has been an important topic of study for several decades; however, its importance has escalated with the signing of the Paris Agreement. One issue that has always hindered implementing sustainability research in practice has been the difficulty in measuring performance. While methods such as life-cycle assessment are available to enable a comparison with alternatives, sustainable performance cannot be related to larger environmental goals. Additionally, such methods often omit uncertainty considerations. The proposed research herein provides foundational measurement science and metrics to bridge the gap between the theories of sustainability and the application. The metrics enable tracking of measurable progress in all aspects of sustainability within a risk-based framework.

This dissertation opens by reviewing and analyzing the literature on sustainability definitions and existing metrics in order to determine the current state of the practice, and to inform the development of the proposed metrics. Next, in order to demonstrate the capacity of risk-based approaches in measuring sustainability performance, a methodology is proposed to calculate the probability of a structure or product meeting sustainability requirements.

Last, the methodology is validated using the National Institute of Standards and Technology's Building Industry Reporting and Design for Sustainability. The validation procedure demonstrated that the methodology was capable of reproducing results from a well-vetted database. The proposed methodology serves as the first step in a "sustainability reliability" metric that is practical, accurate and comprehensive in its coverage.

DEVELOPMENT OF RISK-BASED MEASUREMENTS AND METRICS FOR SUSTAINABILITY QUANTIFICATION OF MANUFACTURED AND CONSTRUCTED SYSTEMS

by

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Foreword

Several chapters of this dissertation contain published work for which the candidate served as co-author. The publications *Sustainability Quantification and Valuation. I: Definitions, Metrics, and Valuations for Decision Making* and *Sustainability Quantification and Valuation. II: Probabilistic framework and metrics for sustainable construction* were published in the *ASCE-ASME Journal for Risk and Uncertainty in Engineering Systems Part A: Civil Engineering Vol 3(3)* with Dr. Bilal Ayyub. The research presented in those journals was conducted primarily by the candidate and the formatting, organization and presentation of that research was primarily the construction of the candidate. The candidate has received the permission of both the co-author of the publications and the director of the graduate program to include their contents in the dissertation. Any chapters that are derived or taken from the previously published work are prefaced with a statement indicating as such.

Dedication

To my family and friends for all of their support over the course of my graduate career.

Acknowledgements

I would like to thank my advisor, Dr. Bilal Ayyub, for his guidance and patience through this process. I am also grateful to my committee members, Dr. Michael Kearney, Dr. Mohammad Modarres, Dr. Peter Sandborn, Dr. Jeffrey Herrmann, and Dr. Klaus Hubacek for their time and service. Portions of this research were funded by the National Institute of Standards and Technology.

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Chapter 1: Introduction

Climate and the environment have come to the forefront of government policy, public demonstration and scientific study, reaching a milestone with the 2016 Paris Agreement. This agreement highlighted a worldwide effort to limit anthropomorphic climate change to under 2° Celsius, while underscoring the important role humanity plays as a steward of the environment. Although the agreement focused on one narrow, though vital, aspect of the environment, it also touched on an increased awareness of how society impacts the environment and the need for the responsible management of it.

Sustainability has been the subject of studies across different fields of science, engineering and public policy for several years. While environmental sustainability is the most well-known, aspects of social and environmental sustainability are also well developed (del Mas Alonso-Almeida 2015). Sustainability has been incorporated into project management (Sanchez et al. 2015), the design and operation of citywide networks (Stokes et al. 2015), as well as a multitude of other topics (Cabezas 2015).

Businesses have also been utilizing sustainability concepts within operations (Herremans 2015) and Reimsbach and Hahn (2015) noted that a failure to consider sustainability can have detrimental effects to the point that self-reporting businesses may not fully disclose negative findings. Kiron and Kruschwitz (2015) noted that sustainability is also tied with resilience; a more sustainable operation being better suited to deal with unexpected negative events.

1.1 Research Goals

Sustainability linkages and associated interdependencies of systems are extensive (NRC 2009). What is problematic is the difficulty in measuring the sustainability of a project in the face of these linkages. Sustainability is a broad category aggregating multiple environmental impacts, complex societal issues and questions of economics. Given the importance of sustainability, in terms of resource preservation, climate change, and resilience, it is vital that a toolset exist to quantify it in a way that allows for meaningful comparisons between projects. Methods do currently exist, though often proving insufficient in simultaneously addressing all aspects of sustainability in a rigorous manner. Life-cycle assessment (LCA) is currently the most popular methodology and focuses on environmental flows to determine the overall impact of a project (a more rigorous discussion of LCA can be found in Chapter 2.4).

Many of the currently available methods neglect social and economic sustainability and the majority do not account for uncertainty. Any major project is a highly planned and tracked process, including inputs from several intermediaries, who themselves may have intermediary suppliers. The process creates a chain of inputs and impacts that are potentially detrimental, in terms of sustainability. LCA methodologies take this into account, but only incorporate environmental impacts and most practitioners assume that the values derived are deterministic (there are reasons for such assumptions, which will be discussed later). The deterministic assumption is flawed conceptually, as any complicated process is bound to contain degrees of uncertainty at various stages. It is here that reliability engineering can provide guidance, by creating a method of quantification that can be used on its own or be readily adaptable to current methodologies, accounting for uncertainty.

In defining the methodology to arrive at a "sustainable reliability" value, there are several questions that must be answered:

1. Which indicators[1] should be included?
2. How are the disparate units of the indicators rectified?
3. How can the methodology maintain flexibility across multiple domains?
4. How can the methodology be extended for use in other calculations?
5. Does the methodology account for all aspects of sustainability?
6. What role do economics play in the methodology?
7. How should uncertainty be considered?
8. Who are the target users, and will the methodology be deemed useful?
9. Does the methodology produce valid results?

Addressing these questions simultaneously is a nontrivial matter. For instance, available indicators are diverse, and relevance may be project-dependent. Converting the disparate units into a consistent unit is required, otherwise no aggregated result can be obtained, and any comparisons can only be done from indicator-to-indicator. The selected unit cannot be arbitrary, otherwise the methodology may not be extendable. This leads to the fundamental question: is it possible to develop a methodology that satisfies the eight questions above, while producing a meaningful result? Due to the difficulty in data collection and the computational requirements, this methodology would be targeted for

[1] An indicator is a measured quantity that informs on the performance or state of an item or system.

high-level decision makers in government or private industry, resolving the first part of question 8. Formally, the objectives of the research are as follows:

1. Develop a methodology for indicator selection that produces, as much as possible, a comprehensive set of indicators describing a project that is flexible enough to account for changes in indicator importance across domains. This will answer questions 1 and 5.
2. Develop the methodology to aggregate all indicators into a single unit of measure, while retaining meaning to a larger, established, calculus that obtains results beyond the final output of the methodology itself. This will answer questions 2, 3 and 4.
3. Ensure that economics and uncertainty are explicitly considered in the methodology, producing a risk-based framework for results. This will answer questions 6 and 7.
4. Produce a practical methodology that simplifies to a process that is understandable and implementable practically. This will answer the last part of question 8 and generate a "sustainable reliability" output.
5. Develop examples and case studies to verify and validate, as much as possible, the methodology. This will answer question 9.

Meeting these objectives will lead to a new methodology for measuring sustainability that can either be used independently or in concert with other methods, such as LCA. Construction and manufacturing were chosen as the areas of interest, although the methodology is general enough to be broadly applicable.[2]

1.2 Organization

The remaining chapters of this paper are organized as follows:

Chapter 2 – A literature review of sustainability definitions, indicator classifications and existing methods.

Chapter 3 – A brief discussion on the use of uncertainty and economics in the methodology.

Chapter 4 – A brief overview of the methods developed, as well as important definitions used throughout the remaining sections.

Chapter 5 – A presentation and discussion of the first methodology developed using Dempster-Schafer structures, as well as examples to illustrate the methodology.

Chapter 6 – Examples developed for the Dempster Shafer method.

Chapter 7 – A presentation and discussion of the second, and preferred, nonparametric methodology, as well as examples to illustrate.

Chapter 8 – A presentation and discussion of the examples developed to validate the methodology.

Chapter 9 – Closing remarks regarding the methodology, findings and potential future research.

[2] For brevity, "item" will be used for the remainder of this paper to refer to both a manufactured product or a structure created through construction.

Chapter 2: Literature Review

This section contains material from the paper *Sustainability Quantification and Valuation. I: Definitions, Metrics, and Valuations for Decision Making* that has been accepted for publication in the ASCE-ASME Journal for Risk and Uncertainty in Engineering Systems Part A: Civil Engineering Vol 3(3) (Webb and Ayyub 2016a).

The literature review focuses on two priorities: (1) to provide guidance for indicator selection within the methodology and (2) to survey the current landscape of methods for evaluating sustainability. Each of these tasks provided a better understanding of the current state of sustainability quantification, as well as historical perspectives on the topic. The review also provided guidance on what an adequate sustainability quantification methodology should contain.

2.1 The Nature of Sustainability Definitions

As a formal concept, sustainability originated in the earth sciences, where it is generally defined as to ability for an ecosystem to "maintain [its] essential function and retain [its] biodiversity in full measure over the long term" (Business Dictionary 2014). This is not the only earth sciences definition, but it does convey sustainability as an ecological concept. Because of the disruptive nature that many human activities have on ecosystems, sustainability has become synonymous with environmental[3] protection, leading to pressure on governments, businesses and engineers to consider environmental impact within operations. The outcropping of these pressures has seen a proliferation of sustainability definitions across fields, industries, businesses and governments. Further

[3] "environment" in this paper refers to the natural environment and ecology. The built environment is considered a separate entity in the bulk of the sustainability literature.

efforts have since added societal and economic sustainability issues to the original environmental definitions.

A taxonomy proposed by Dimitrov (2010) is presented in **Error! Reference ource not found.** The taxonomy provides an entry point to understanding the diversity of sustainability definitions and what elements might be included in a working definition. The key findings from Figure 0-1 are the use of the environmental definition at the highest level, thus reserving it as the goal for any sustainability definition, and the use of four secondary branches. These four branches provide context for the vital dimensions of sustainability: societal, economic, political and engineering. Political and engineering sustainability are typically not included in sustainability (see Figure 0-2) considerations explicitly, but play an important role in the implementation of sustainability goals.

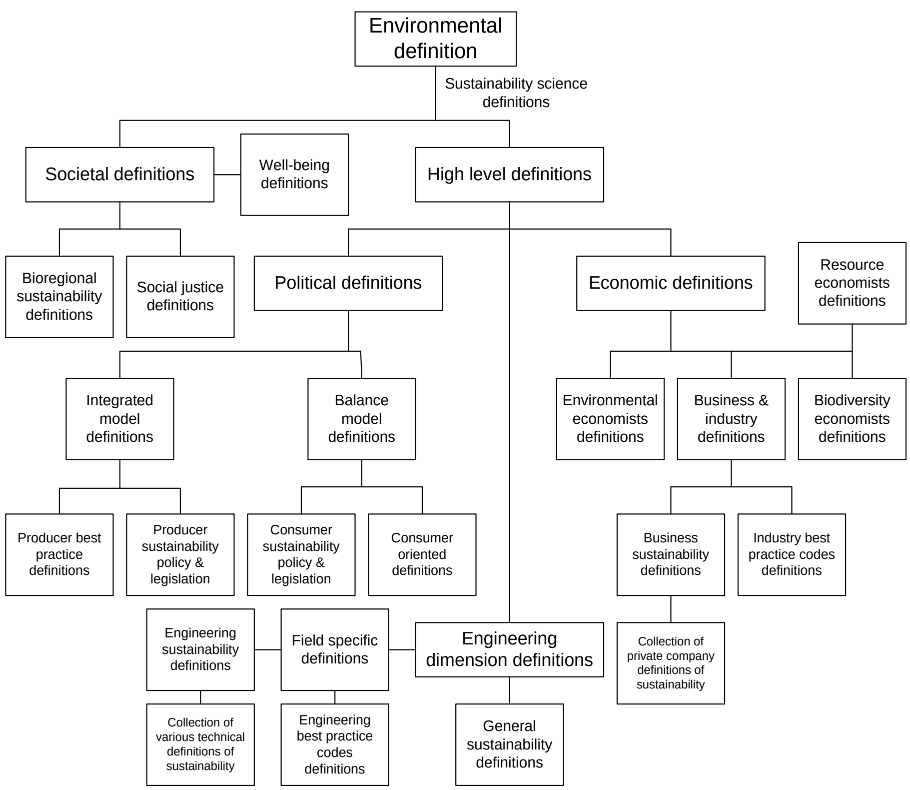


Figure 0-1. Hierarchical relationship among the definitions of sustainability

Understanding the basics of sustainability requires establishing what it is, thus the first step to developing the proposed methodology is finding, or generating, a suitable definition. Classifying definitions will guide what a proper definition should include. Figure 0-2 illustrates the classic view of sustainability as a triad of considerations, often referred to as the "three pillars" of sustainability. The central overlap where all three pillars converge is the focus of comprehensive sustainability, in that all aspects under the three pillars are considered. The "three pillars" view is widely accepted and used in the concept of the "triple bottom line," an economic framework for valuing sustainability in business practices (EPA 2013b). As an economic framework, the triple bottom line exists within the larger context of laws, regulations and societal needs or desires. There is also an invalid assumption that the economy exists outside of society, or that society exists outside the environment.

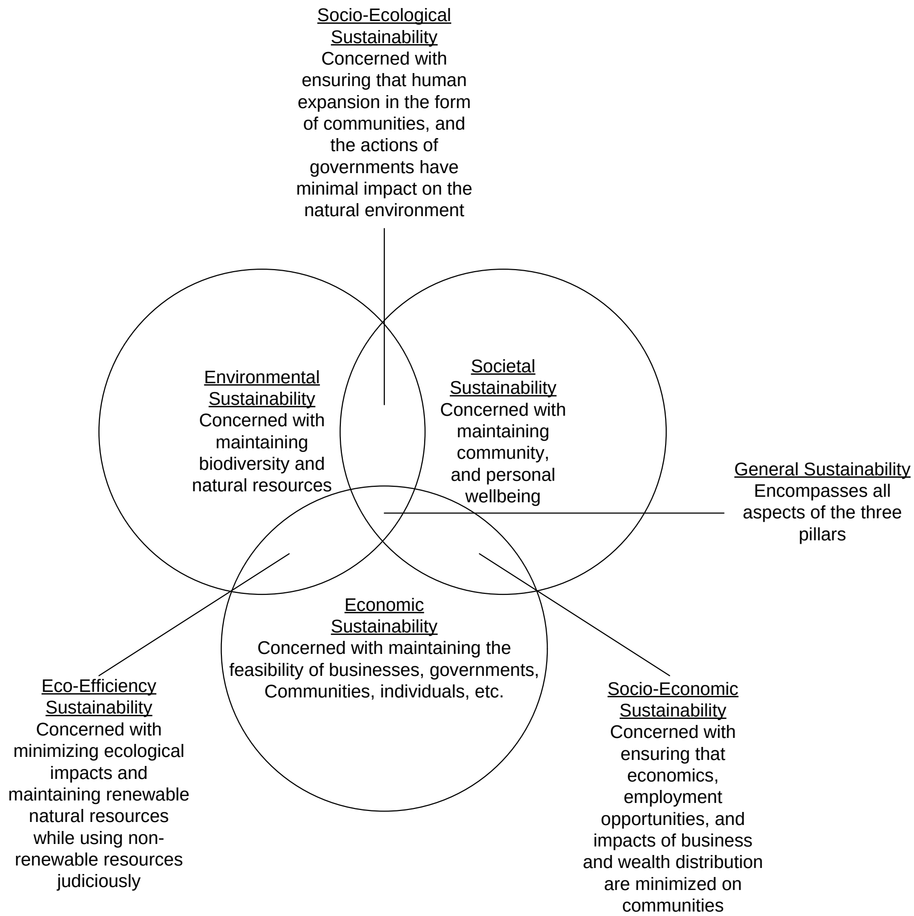


Figure 0-2. Venn diagram of the three pillars approach to sustainability

The first step to generating a more realistic view of sustainability is to understand the relationships between the three pillars, and the additional dimensions from Dimitrov (2010). By tracing how sustainability efforts are driven from one dimension to the next, a better model of sustainability can be derived, from which a "good" sustainability definition can be created. Figure 0-3 presents the nature of the drivers toward sustainability and the elements that each level adds to the requirements for a working definition.

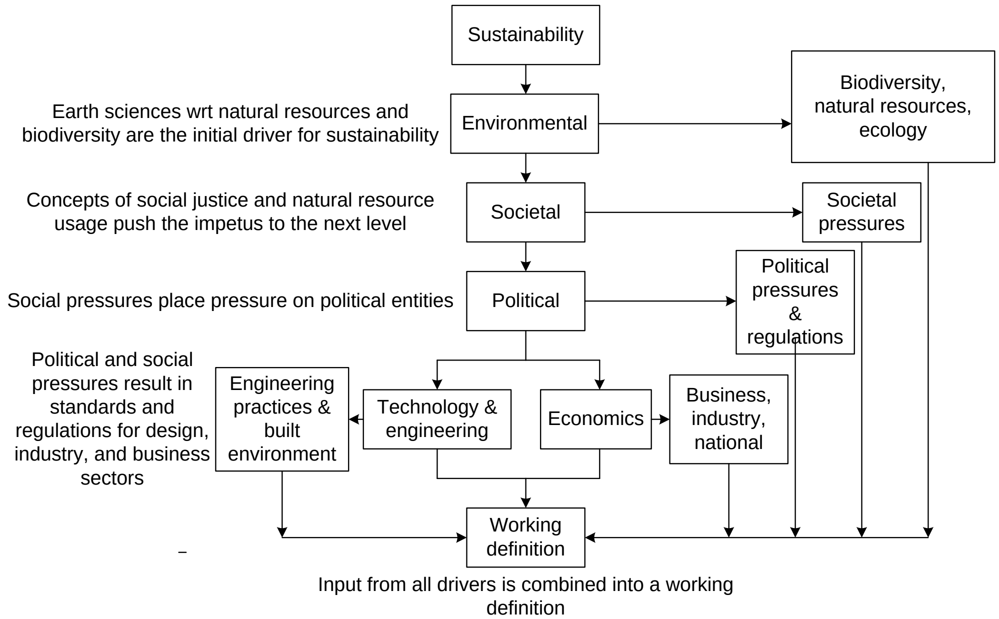
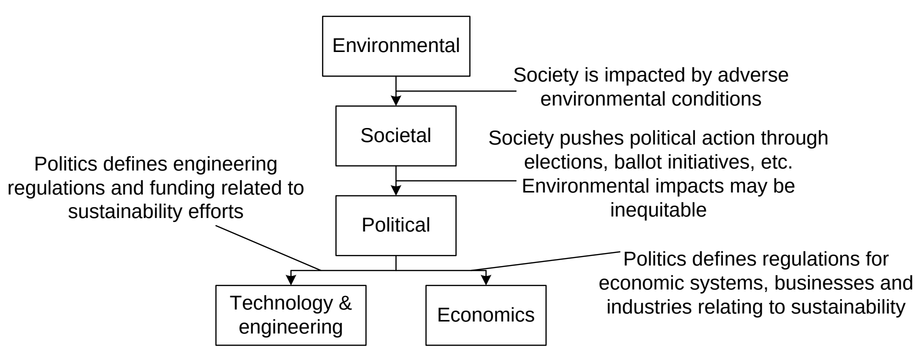
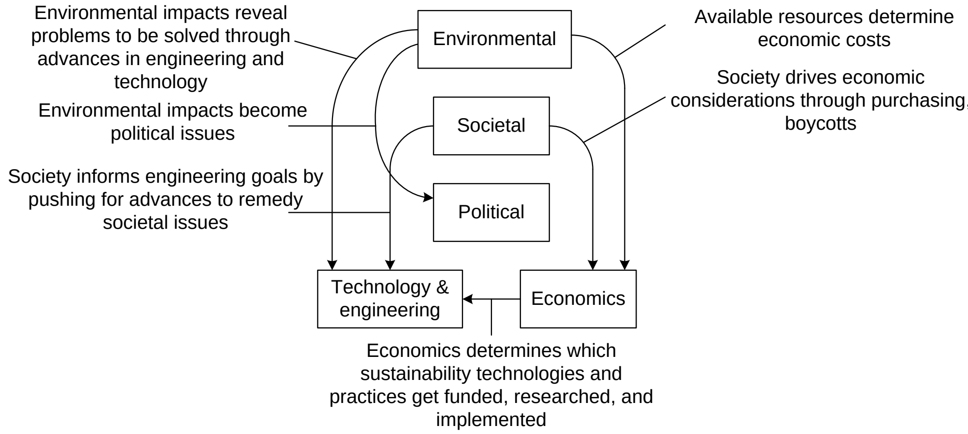


Figure 0-3. Tree structure of sustainability drivers towards a working definition

The hierarchy in Figure 0-3 illustrates that each level of sustainability adds further requirements and limitations to a working definition. What Figure 0-3 does not represent is a strict hierarchal nature that all sustainability efforts must follow. Motivation for, and pushback to, any sustainability effort could theoretically begin at any point in the hierarchy and result in feedback loops or jumping multiple, as illustrated in Figure 0-4 and Figure 0-5.

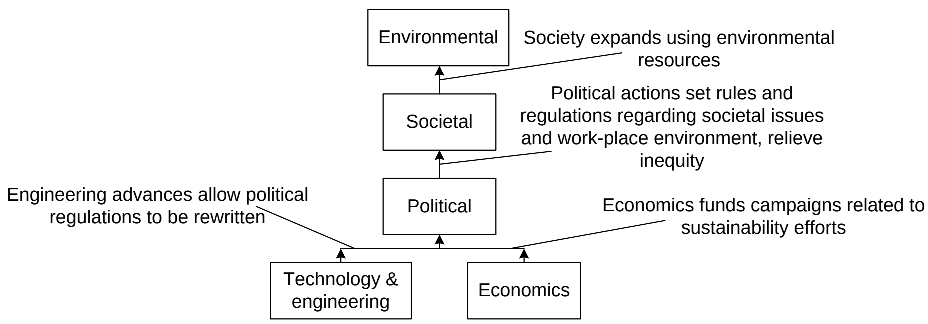


a) Direct relationships

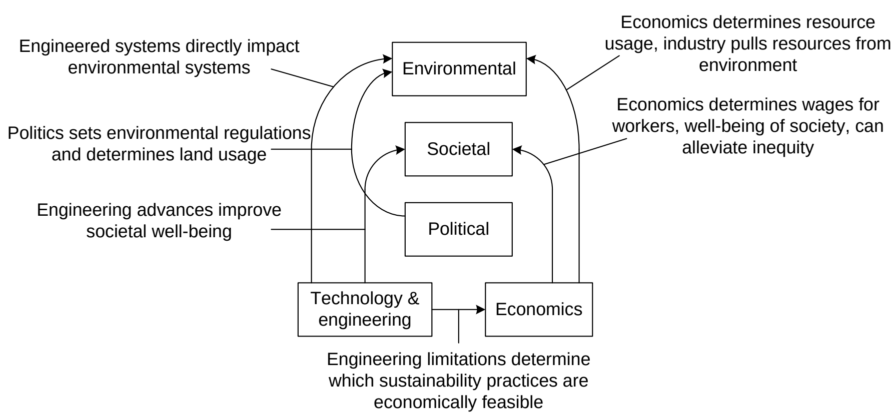


b) Bypass relationships

Figure 0-4. Downward hierarchical relationships between selected sustainability drivers



a) Direct relationships



b) Bypass relationships

Figure 0-5. Upward hierarchical relationships between selected sustainability drivers

An attempt to address the issues with the three pillars Venn diagram was made, based on the additional dimensions suggested by Dimitrov (2010) and the hierarchies in Figure 0-3, Figure 0-4 and Figure 0-5, as presented in Figure 0-6.

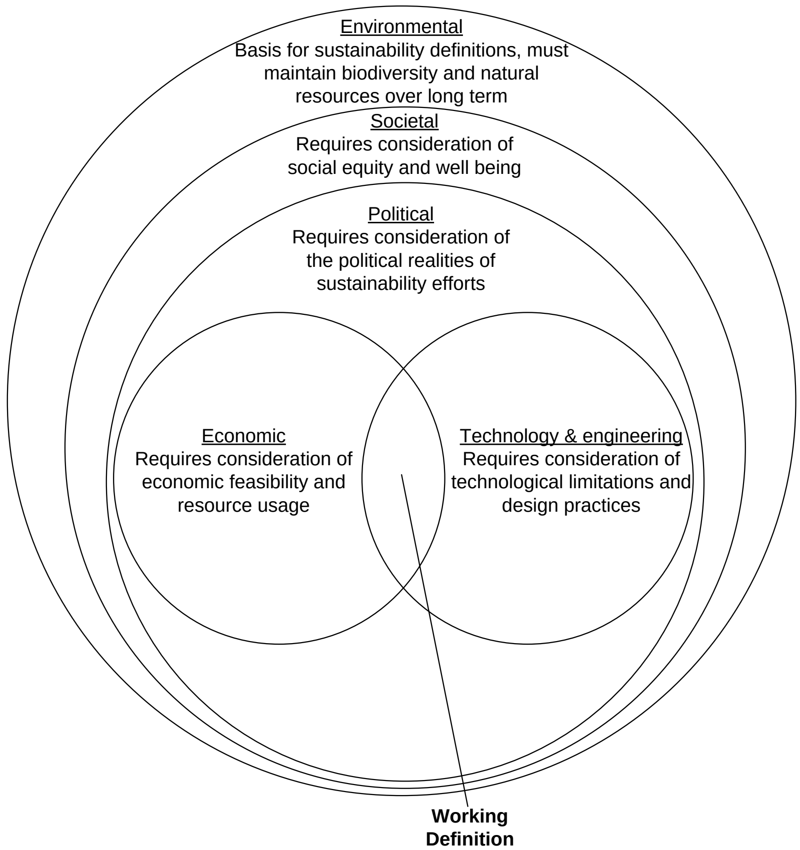


Figure 0-6. Nesting and intersections needed for sustainability definitions based on sustainability drivers[4]

In the conceptual model established in Figure 0-6, each nested dimension is constrained by the larger sphere(s) in which it exists. Therefore, all dimensions are constrained by the environment, requiring it to be the initial basis for any definition of sustainability. Use of a nested structure also recognizes that each dimension of sustainability adds limitations, barriers or opportunities that affect the level of sustainability that can be achieved, while preserving the relationships in Figure 0-4 and

[4] The size of the circles and the overlap have no meaning.

Figure 0-5. This is through the ability of each nested dimension to “push outward” against any dimension it is contained within, or to “push inward” towards any dimension contained within it. This ultimately prescribes the requirements that a complete sustainability definition should include:[5]

1. Consideration, at minimum, of all three pillars of sustainability, and preferably, the additional dimensions of politics, engineering, technology and science.
2. A realization that sustainability efforts are constrained by the nature of the other dimensions impacted by any action. Examples include business viability, political will, public perceptions, governmental limits to power or limitations on available technology.
3. An understanding that different industries may have different sustainability needs or goals. This represents the implicit subset of the economic and engineering dimensions of sustainability, based on the specific nature of the sustainability effort and it includes evaluation of the total lifecycle.

2.2. Sustainability Definitions

Having determined a criterion for selection, an extensive literature review of academic, public and industrial definitions of sustainability was conducted. Three categories were used: (1) general, (2) construction and (3) manufacturing. General definitions were those that, instead of pertaining to a specific engineering field, defined sustainability in a broader context. Construction and manufacturing definitions narrowed

[5] The working definition here is developed for sustainability applications. The evaluation of definitions in this paper does not presume that broader definitions are inherently flawed or wrong, only that those definitions do not suit the immediate needs of this paper.

the broad context of general definitions to the respective disciplines. In evaluating definitions, preference was given to scholarly or regulatory sources.

In examining the multitude of sustainability definitions, another requirement was determined. Brundtland report's definition of sustainability was "the ability to meet the needs of the present without compromising the ability of future generations to meet their own needs" (Keeble 1988). A working definition should thus also include a fourth feature: consideration of future generations' needs.

Using the guidelines above, candidate working definitions were selected, which can be found in Table 0.1, along with the reasoning for selection. Some of the definitions did not meet all the guidelines, but were chosen because each was used when determining policy at a national level. The definitions in Table 0.1 serve as the starting point for generating sustainability metric and indicator criteria.

Table 0.1. Selected sustainability definitions

Context	Definition and Source	Reasoning
General	“Creating and maintaining conditions under which humans and nature can exist in productive harmony and that permit fulfilling social, economic, and other requirements of present and future generations” (EPA 2012).	The EPA general definition is based on the National Environmental Policy Act and informs high level policy decisions.
General	“Sustainable development is the challenge of meeting human needs for natural resources, industrial products, energy, food, transportation, shelter, and effective waste management while conserving and protecting environmental quality and the natural resource base essential for future development” (ASCE 2009).	The ASCE definition provides a better basis for metrics than the EPA definition and meets the suggested guidelines of the paper.
Construction	“The practice of increasing the efficiency with which buildings and their sites use and harvest energy, water, and materials and protecting and restoring human health and the environment, throughout the building life-cycle: siting, design, construction, operation, maintenance, renovation and deconstruction. The practice of creating and using healthier and more resource-efficient models of construction, renovation, operation, maintenance and demolition” (EPA 2012).	The EPA construction definition does not explicitly consider economic sustainability. However, it informs high level policy and must be considered.
Construction	"Sustainable building may be defined as building practices, which strive for integral quality (including economic, social and environmental performance) in a very broad way. Thus, the rational use of natural resources and appropriate management of the building stock will contribute to saving scarce resources, reducing energy consumption (energy conservation), and improving environmental quality” (Sassi 2006).	The definition from Sassi (2006) includes explicit consideration of all three sustainability pillars and serves to enhance the EPA definition’s lack of economic consideration. It does lack explicit reference to the full life-cycle of the building though.
Green Buildings	“The practice of increasing the efficiency with which buildings and their sites use and harvest energy, water, and materials; and protecting and restoring human health and the environment, throughout the building life-cycle: siting, design, construction, operation, maintenance, renovation, and deconstruction” (EPA 2016).	The EPA green building definition is included, as it defines the goal of a “green” building from a policy standpoint. As with the EPA construction definition, it excludes the economic pillar.

Table 0.1 is not exhaustive, and many definitions exist, or could be constructed, that meet the above guidelines. For instance, most professional organizations and corporations concerned with sustainability have unique definitions. The American

Society of Civil Engineers (ASCE) defines sustainability as "a set of economic, environmental, and social conditions in which all of society has the capacity and opportunity to maintain and improve its quality of life, without degrading the quantity, quality, or the availability of natural, economic, and social resources" (ASCE 2013). The ASCE definition is just as applicable as any presented in Table 0.1, especially considering it is widely respected in industry. For brevity, not all definitions evaluated are included in this section, but many can be found in Appendix A.

2.3 Sustainability Indicator Classifications

Sustainability metrics, indicators and measures are numerous, making categorization essential to understand what each indicator is informing on.[6] It also helps to ensure that all important aspects of a system are evaluated. The International Organization of Standards (ISO) provides a broad classification scheme that divides environmental indicators, the largest class of sustainability indicators, into performance and condition indicators, as shown in Figure 0-7. The former is a measure of system behavior, while the latter describes the state of a system (Sikdar 2003). A broad classification establishes what an indicator should do, and a narrower classification is necessary to tailor indicators to the needs of practitioners.

[6] In this paper, sustainability indicators include all values informing environmental performance, societal issues related to sustainability or economic performance related to sustainability. Indicators are measurable and can be estimated or modeled. Thus, values such as environmental flows from an LCA analysis may be considered indicators, because of measuring environmental performance.

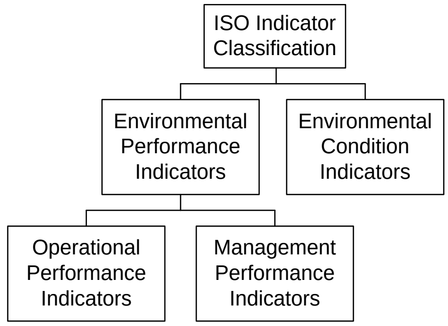


Figure 0-7. ISO environmental indicator classification

Most classifications use the three pillars as the highest-level grouping and further refine categories from there. Boyd (2006) suggested using institutional components as an additional grouping. Hanley et al. (1999) broke the three pillars down into type and group. Type refers to whether the indicator is a single value or an aggregated value, while group refers to a sub-area of the pillar it resides in.

The five capitals approach, though not explicitly an indicator classification method, breaks capital into human, social, natural, manufactured and financial groups (Viederman 1996). The general organization of the five capitals, shown in Figure 0-8, is akin to Figure 0-6, except the framework keeps the three pillars as a separate principle and focuses on what is required to create a good or service. However, such a framework would be adaptable to an indicator classification system.

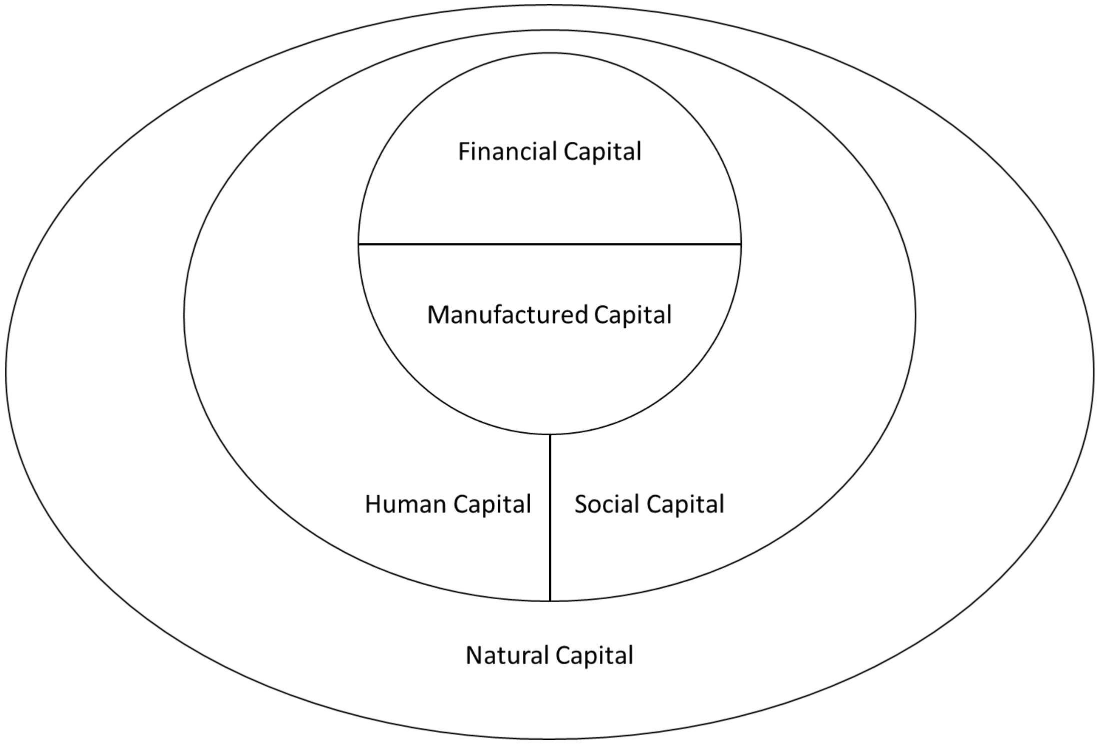


Figure 0-8. One variant of the five capitals concept

The natural step framework is not based on the three pillars concept. Instead, it defines sustainability as a society where "nature is not subject to increasing (1) concentrations of substances from the earth's crust … (2) concentrations of substances produced by society … (3) degradation by physical means … [and] (4) in that society there are no structural obstacles to people's health, influence, competence, impartiality, and meaning" (The Natural Step 2016). The four points listed in the definition of sustainability are used to classify indicators by what those indicators impact.

The Environmental Protection Agency (EPA) has a system developed by Fiksel et al. (2012), which has multiple levels of classification. Table 0.2 summarizes the EPA indicator taxonomy levels. The EPA taxonomy is thorough in describing an indicator in terms of the EPA's Report on the Environment (ROE) sections: air, water, land, human exposure and health and ecological condition (Fiksel et al. 2012). Notably, it does not consider economic sustainability. The highest level, Scale, is a reference to the geographic scale of the examination (global, national, etc.). Country/Organization references the entity using the indicator and Pillar relates the indicator back to one of the three pillars. The EPA ROE topic, Office of Research and Development (ORD) Program and the Triple Value categorization are EPA specific. The lowest level, Dimension, reports how many pillars the indicator can possibly inform on (Fiksel et al. 2012).

Table 0.2. EPA indicator taxonomy

Level	Description
Scale	Level at which sustainability is being examined (global, national, etc.)
Country/Organization	Specific region or organization that is being examined
Pillar	Which of the three pillars it informs
EPA ROE Topic	Area of the EPA Report on the Environment that the indicator falls under (Fiksel et al. 2012)
Program	Program under the EPA Office of Research and Development (ORD) that the indicator would provide information on
Triple Value (3V)	ORD framework that classifies indicators into Adverse Outcome (AOI), Resource Flow (RFI), System Condition (SCI) or Value Creation (VCI)
Dimension	The pillars that are affected (1D, 2D or 3D)

Indicator classifications specific to manufacturing and construction are also available. The Lowell Center for Sustainable Production (LCSP) breaks production indicators into five levels, as shown in Table 0.3 (Veleva et al. 2001). This system focuses on facility effects and the supply chain, indicating a long-term view of sustainability; however, it neglects end of life considerations. It emphasizes the three pillars as well as government regulation compliance.

Table 0.3. LCSP levels for sustainability

Level	Description
1. Facility Compliance and Conformance	Focuses on regulation compliance (Veleva et al. 2001)
2. Facility Material Use and Performance	Focuses on material input, output, efficiency and performance (Veleva et al. 2001)
3. Facility Effects	Focuses on the potential effects of a facility on the environment, workers, public health and communities (Veleva et al. 2001)
4. Supply Chain and Product Lifecycle	Focuses on supply chain and product distribution (Veleva et al. 2001)
5. Sustainable Systems	Brings individual production into the broader sustainability network (Veleva et al. 2001)

Construction indicators were developed by Fernandez-Sanchez et al. (2010). A process was proposed to determine a classification scheme (Table 0.4), as well as a final set of indicator classifications (Table 0.5). The final classification scheme was broken down along the three pillars, while including broad coverage of indicator topics. The process to arrive at Table 0.5 was composed of data review and analysis and expert elicitation (Fernandez-Sanchez et al. 2010).

Table 0.4. Steps for classification of indicators for construction, adapted from Fernandez-Sanchez et al. 2010

Step	Description
1. Identification	Various techniques (brainstorming, analysis, literature review, etc.) used to identify sustainability opportunities
2. Classification	Classification of opportunities into three pillars
3. 1st Prioritization	Analysis-based prioritization
4. 2nd Prioritization	Expert judgment-based prioritization
5. Indicator Set	Final indicators to be used based on prioritization

Table 0.5. Indicator classification, adapted from Fernandez-Sanchez et al. (2010)

Pillar	Subcategories
Social	Culture, accessibility, participation, security, public utility and social integration
Environmental	Soil, water, atmosphere, biodiversity, resources and energy
Economic	Costs, technical requirements, bureaucracy, social economy and heritage

Some indicators may be suitable across nearly all items, while others are irrelevant for a particular item, or class of items. The possibility of superfluous indicators means that a rigid classification scheme may not be suitable. Instead, a scheme that is broad enough to cover all pertinent areas, while allowing for refinement to only the required indicators is preferable for the methodology proposed here. This is a similar finding to Boyd's (2006).

For the proposed methodology, a variation on the EPA classification was chosen.[7] The proposed taxonomy was tailored to the construction (Table 0.6) and manufacturing domains (Table 0.7). The EPA-specific levels were replaced with a generic "aspect" level to define what the indicator informs on; for instance, human health or water quality. The "phase" level indicates what lifecycle phases the indicator applies to. The "triple value" level may be retained, because it still applies without the ORD level, however it is not required. The "pillar(s)" level combines the "dimension" and "pillar" levels from Fiksel et al. (2012) into a single level.

[7] This does not imply that Fiksel et al. (2012) or the proposed variation are the best suited for all or any options.

Table 0.6. Proposed indicator classification for construction

Level	Description
Scale	Level at which sustainability is being examined (global, national, etc.)
Country/Organization	Specific region or organization that is being examined
Pillar	Which of the three pillars it informs
Aspect	Materials, energy, water, air, financial, community and culture, health, jobs and others, as required
Phase	Design, construction, operation, maintenance, decommissioning and demolition
Triple Value (3V)	EPA Office of Research and Development framework that classifies indicators into adverse outcome index (AOI), resource flow index (RFI), system condition index (SCI) and value creation index (VCI)

Table 0.7. Proposed indicator classification for manufacturing

Level	Description
Scale	Level at which sustainability is being examined (global, national, etc.)
Country/Organization	Specific region or organization that is being examined
Pillar	Which of the three pillars it informs
Aspect	Materials, energy, water, air, financial, community and culture, health, jobs and others as required
Lifecycle Phase	Design, supply chain, production, useful life, decommissioning and disposal
Triple Value (3V)	EPA Office of Research and Development framework that classifies indicators into adverse outcome index (AOI), resource flow index (RFI), system condition index (SCI) and value creation index (VCI)

2.4 Sustainability Metrics

One of the most popular methods to determine environmental performance is LCA. Although not technically a sustainability metric, the goal of LCA is to facilitate environmental impact comparisons with alternatives; its methodology produces resource flows that could be used to inform on sustainability. The National Institute of Standards and Technology (NIST) utilizes LCA in the Building Industry Reporting and Design for

Sustainability (BIRDS) and the Building for Environmental and Economic Sustainability (BEES) online tools (Lippiatt et al. 2010, Kneifel et al. 2016). LCA experts are prevalent throughout the industry. The general process involves taking a comprehensive inventory of all environmental flows[8] related to an item at every phase of its lifecycle. Using expert elicitation and complex calculations, an aggregated environmental impact score is then obtained. This score is used to compare to other options; however, it lacks any real meaning when isolated. While widely accepted, there are issues with LCA. It requires extensive data collection, expert judgement and its results may be method-dependent. It still represents the state of the art though, for measuring the environmental impact of engineered products. Although LCAs do not explicitly account for financial impact, a financial analysis can be used alongside the LCA results. Social impacts are generally omitted, due to issues with obtaining, using and interpreting the pertinent data.

Standard LCAs do not account for uncertainty, although some risk-based approaches have been developed (Ayoub et al. 2014, Anex 200). Padgett and Li (2014) incorporated uncertainty from traditional sources in construction as well as a sustainability analysis itself. The scope was limited to hazard resistance of a structure, which is not a generally applicable approach. Anex (2000) proposed a more generalized approach using probabilistic inventories along with a Bayesian risk-based technique to account for uncertainty.

While LCA is an extremely popular means to measure environmental impact, there is a plethora of sustainability indicators, indices and metrics available; more than

[8] An environmental flow is a measure of the amount of a pollutant, or the equivalent amount of a standard pollutant if more than one is responsible for a particular type of environmental degradation, consumed in any part of the lifecycle.

can be fully reviewed herein. There is little consistency in what the metrics cover or include. Most focus on only one pillar, typically the environment, and use a small number of set indicators, while others attempt a more complete coverage of sustainability issues. One consensus, however, is that the societal pillar is the hardest to measure, due to difficulties in quantifying human behavior, social capital and human health (Stevens 2005). Although valid concerns, some indicators have been developed for the social pillar (Moldan 2012, Darby 2006, Dempsy 2011, Dillard 2008, Magus 2010, S. McKenzie 2004). Most forego the societal pillar, due to difficulties in obtaining reliable values.

Another popular class of metrics are the product sustainability indices (PSI). These mainly focus on manufactured goods, although the concepts are transferrable across domains. Comprehensive lifecycle PSIs that use the results of a traditional LCA were developed in Jawahir et al. (2006), Zhang et al. (2012) and Shuaid et al. (2014). The LCA results were ranked on a zero to ten scale based on how well a product achieved sustainability in a particular phase and pillar. These scores were aggregated to a final PSI represented by a probability. This methodology has the benefit of producing results for intermediate steps, such as lifecycle phases and relating each "to [the] actual sustainability content in the product" (Zhange et al. 2012). It also used the results of an LCA, which are widely accepted, and considered uncertainty, though roughly.

Many examinations of sustainability rely on measuring the actual performance as compared to some target performance. A common technique for this is a spider diagram, an example of which is found in Figure 0-9 for a hydroelectric plant. The figure is adapted from an example supplied by the Hydropower Sustainability Assessment Protocol (2014). The benefit of a spider diagram stems from its ability to efficiently display how well an item compares to a set of targets, or another item, across multiple indices. The axis can be a rank indicator, an actual value or ratios of target to measured value. A spider diagram cannot aggregate these measures into a single value for comparison though.

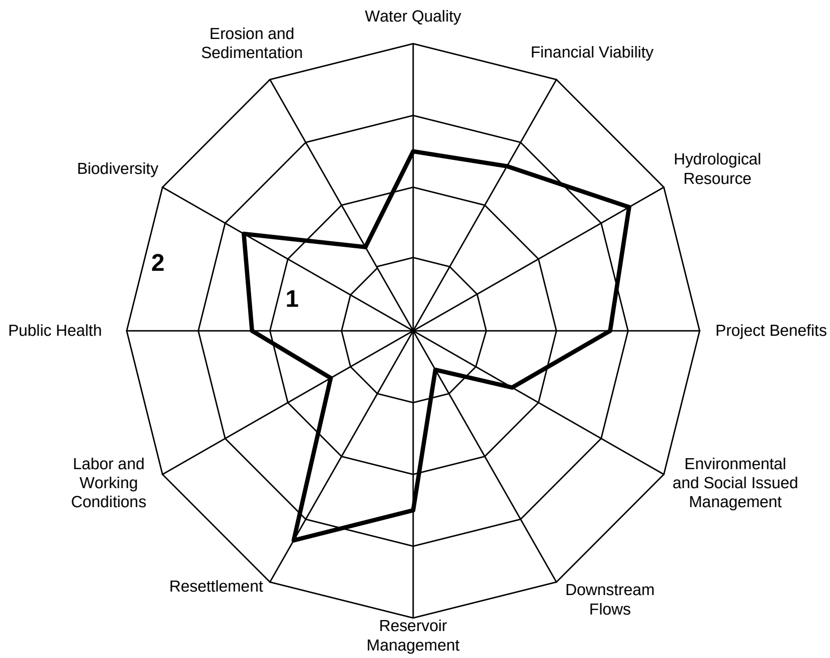


Figure 0-9. Spider diagram for hydroelectric plant (note: numeric values indicate the ratio of actual indicator value to a target value)

Uncertainty consideration in a spider diagram is not widely used. This is due to the difficulty in adding the uncertainty information visually without making the diagram cluttered.

Chapter 3: The Importance of Uncertainty and Economics

This section contains material from the paper *Sustainability Quantification and Valuation. I: Definitions, Metrics, and Valuations for Decision Making* that has been accepted for publication in the ASCE-ASME Journal for Risk and Uncertainty in Engineering Systems Part A: Civil Engineering Vol 3(3) (Webb and Ayyub 2016a).

The information required to accurately assess the sustainability of a complex system is significant. The data are usually provided as deterministic values, despite being the result of an uncertain estimate, such as expert elicitation, that would result in questionable accuracy. There is often little that can be done when obtaining data from outside sources, meaning the values may be all that is available. However, ignoring the compounding uncertainty in using these values may lead to overall results that have a great deal of uncertainty.

3.1 Types of Uncertainty

Uncertainty can be broadly defined as a lack of complete knowledge. In an engineering context, uncertainty is generally viewed as "knowledge incompleteness due to inherent deficiencies in acquired knowledge" (Ayyub and Klir 2006). It can be further broken down into aleatory and epistemic uncertainty. Aleatory uncertainty is any knowledge deficiency caused by inherent randomness or something that is nondeterministic by nature. This type of uncertainty is often associated with variables related to the physical world (wind loading, seismic loading, etc.) and cannot be reduced by acquiring more data. At best, the distributions quantifying the uncertainty in such variables can be better defined. Epistemic uncertainty is a lack of attainable knowledge. In theory epistemic uncertainty can be reduced by improving the state of knowledge through research, or other forms of knowledge acquisition. In reality, reducing epistemic uncertainty may prove infeasible due to a lack of resources, a lack of technological capacity or ethical, legal or sociopolitical constraints. Beyond aleatory and epistemic uncertainty, there are further taxonomies. A complete definition of such taxonomies is outside the scope of this paper, but it can be found in (Ayyub and Klir 2006).

3.2 Data Sources and Knowledge Reliability

It is important to understand that uncertainty exists in nearly all data sources and any knowledge gained from data sources will not necessarily be reliable. Scholarly sources of data that have been peer-reviewed and verified are generally more reliable than anecdotal evidence for instance. Expert elicitation can be a reliable data source, but that reliability is highly dependent on the problem formulation, the chosen experts and the means by which the elicitation was conducted. Therefore, the reliability of expert elicitation, no matter how accurate the ultimate result, is always in doubt due to the compounded uncertainty of a source that is a combination of experience, expertise and subjectivity.

Even direct observation may not be reliable for the purposes of a task. Consider the Colorado River Compact. When it was first drafted in 1922, direct observation put the flow of the river at 16.5 million acre-feet (maf) based on available Reclamation Bureau data and this value was written into the compact. In actuality, the average river flow using historical data was 13.5 maf, with the volume of water being highly erratic, ranging from as low as 4.4 maf to 22 maf (Gelt 1997). Basing the compact on a high flow value has led to issues in water allocation between the states in the compact over many years.

3.3 Uncertainty and its Role in the Methodology

To illustrate the role and the effects of uncertainty, consider the National Climate Assessment (NCA) forecasts of climate change and carbon reduction efforts. Carbon emission reduction goals were set based on the NCA and other sources of input. The NCA forecasts contain prediction bounds indicating uncertainty over time. When a carbon emission reduction goal is set, it deterministic, meaning the carbon emission reduction goal is based on an uncertain value. If the NCA used the lower bound of the prediction interval, then the carbon emission reduction goal will be lower, but with a higher probability of it being exceeded. Carbon reduction efforts would not be as strong, but if a higher value was used, the climate change goal would likely fail. If the upper bound was used, more stringent goals would be set that are less likely to be exceeded, however it would likely mean a larger negative impact on the economy, to the point of possible infeasibility. In making such decisions, whatever value is proposed carries with it an implicit probability that it will go too far, or not far enough. For the purposes of setting an achievable goal, the explicit consideration of uncertainty is removed, but the reality of that uncertainty remains.

To aid in making decisions like the one above, a toolset that helps simplify the probabilistic nature of the problem without negating it is important. This means that any final reported value is based on the probability of success, and not a deterministic approach. For a situation with a small number of variables, the calculation may not be overly difficult, but for a complex item with multiple inputs, phases and a long lifespan, the problem is nontrivial.

The goal of the proposed research is to utilize the concepts behind various quantification methods to derive a methodology that links sustainability quantification and uncertainty in a risk framework. As the performance of buildings and products are difficult to measure, especially in the forward-looking manner that sustainability requires, the methodology must account for the lack of knowledge that exists. Uncertainty, risk and sustainability are inextricably linked, and the proposed methodology was developed under such an assumption.

3.4 Economics and Consequence Valuation

In discussing sustainability quantification, economics plays a central role. Although the primary goals of sustainability are environmental protection, economic longevity and social equity, the reality is that private and public organizations are inevitably going to play a large role in enacting such measures. Private organizations are typically guided by shareholder concerns, while public entities are often limited by funding, politics or public opinion. Omitting these realities or trivializing these concerns can increase organizational inertia among private organizations, or it can leave public organizations underfunded and ineffective.

The term “economics of sustainability” generally refers to the practice of incorporating natural resources and societal costs directly into economic models (Ikerd 1997). While a full discussion regarding such a methodology is warranted, it is outside the scope of this research. Economics, in the context of the proposed methodology, will focus on the notion of consequences. In short, a consequence is a monetary loss or gain resulting from an inherently risky or uncertain situation. This allows for both favorable and negative outcomes to be considered.

Chapter 4: Proposed Methodology Development

This section contains material from the paper *Sustainability Quantification and Valuation. I: Definitions, Metrics, and Valuations for Decision Making* that has been accepted for publication in the ASCE-ASME Journal for Risk and Uncertainty in Engineering Systems Part A: Civil Engineering Vol 3(3) (Webb and Ayyub 2016b).

4.1 Impetus

As previously noted, there has been little effort made to create a comprehensive accounting of uncertainty in an effort to quantify sustainability. This is troubling, given the use of uncertain sources of information to develop metrics. When a deterministic value is reported the uncertainty does not vanish, but without acknowledging the probabilistic nature of any reported value, the uncertainty is trivialized. To address this issue, the proposed methodology attempts to bring all values into a probabilistic framework. This accounts for the uncertainty and allows for extended calculations, since probability is universal beyond a single metric.

4.2 Important Definitions

To understand the methodology, several terms must be defined, as below.

Indicator distribution – A probability distribution of the indicator-level performance of an item.

Target distribution – A probability distribution of the indicator level required to meet a sustainability target. This is the means through which sustainability is quantified. An indicator or a resource flow from an indicator distribution sets the level. The target distribution translates that level into a likelihood of achieving sustainability.

Consequence function – A functional mapping the relates indicator level to a consequence. This could also be represented by a probability distribution if desired.

Need probability – The probability of achieving a sustainability target. This is an exceedance probability calculated from a target distribution using a set indicator level.

Consequence – A financial outcome from achieving, or failing to achieve, a sustainability target.

Investment – The lifecycle cost of all investments in the item.

Benefit – The expected benefits derived from the item, excluding any consequences derived from the item.

It is important to note that the indicator and target distributions, while both distributions of the indicator level, are inherently independent. The relationship is akin to that between a probability distribution of spectral acceleration for an earthquake of assumed magnitude and the fragility curves of a structure. A spectral acceleration simulated from the distribution is merely a prescribed (or set) applied load as far as the fragility curve is concerned. In the same way, the simulated value from an indicator distribution merely produces a “load” that the target distribution uses to calculate an exceedance probability.

4.3 Overview of Methods

Two methods are introduced to develop a sustainability measure under uncertainty. The first uses Dempster-Shafer structures (DSS), which are discussed in full in subsequent sections. In practice, DSS are impractical and unwieldy, but for this analysis DSS serve as a good illustration of the nature of uncertainty, and why it must be addressed. The second method involves the use of non-parametric techniques. This methodology is more intuitive and practical. The results of the methodology may serve as a sustainability metric or each may be further simplified into a less comprehensive metric.

Chapter 5: Dempster-Schafer Structure Method

This section contains material from the paper *Sustainability Quantification and Valuation. II: Probabilistic framework and metrics for sustainable construction* that has been accepted for publication in the ASCE-ASME Journal for Risk and Uncertainty in Engineering Systems Part A: Civil Engineering Vol 3(3) (Webb and Ayyub 2016b).

5.1 Dempster-Shafer Method Steps

This section introduces the basic steps of the proposed methodology. Figure 0-1 displays the procedure and each step is described in the subsequent sections.

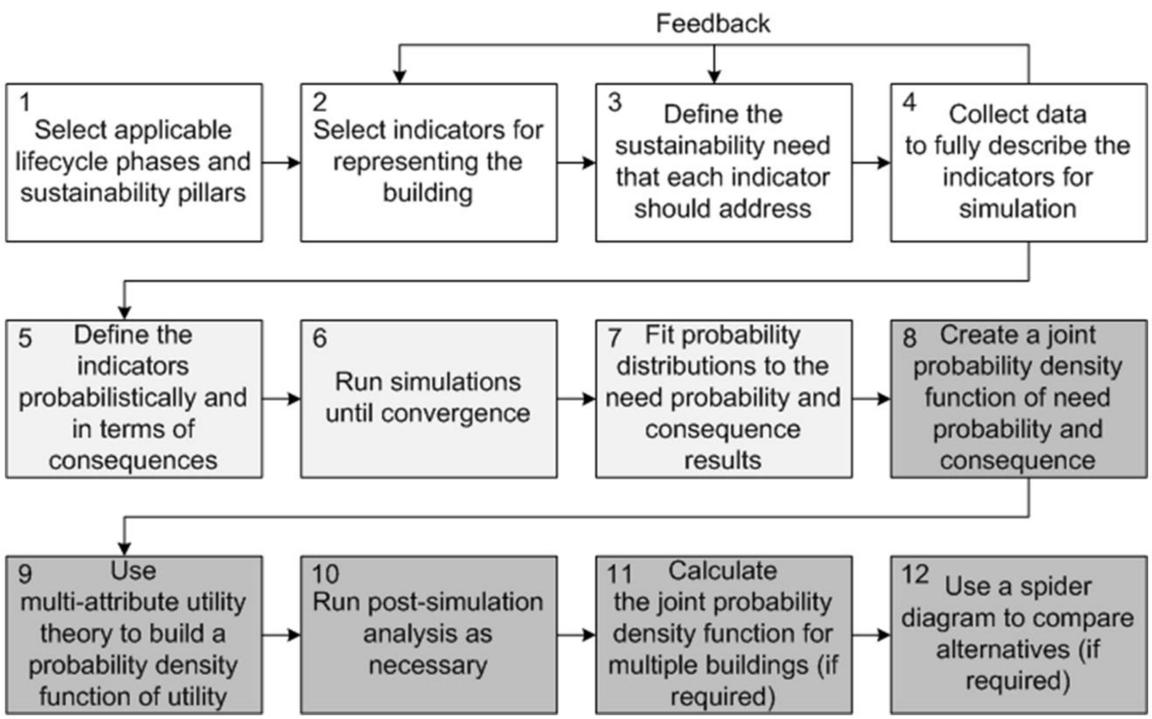


Figure 0-1. Steps of proposed methodology. White - analysis specification. Light-grey - simulation. Dark-grey - postprocessing and analysis

5.1.1 Select Applicable Lifecycle Phases and Pillars

Prior to the numeric calculations, the required lifecycle phases and sustainability pillars for the analysis should be identified. In general, all sustainability pillars apply. The choice of the lifecycle phases should be based on the specific item in question. For example, an existing structure will not always require considering the design or construction phases, while for a new structure all phases would be relevant.

5.1.2 Select Indicators

Having a comprehensive set of indicators based the needs of all stakeholders enables a full representation of the sustainability impact of the structure or product being analyzed. Indicator selection is dependent on the item of interest, making it difficult to generalize. The varied sustainable targets make the identification of a collectively exhaustive and minimal set of indicators infeasible for applicable across all industries and sectors without any irrelevant indicators. Such a set may exist for certain sectors or industries. In all cases, indicators are based on the individual project under analysis. Expert opinion elicitation can provide the initial indicator scope if a standardized set is unavailable. Screening and judgment can help to remove any inapplicable indicators, conceded indicators[9] or those having negligible impacts. The steps involved in determining indicators contain feedback loops as the indicators are filtered down to a final set for analysis.

Ideally, all indicators focus on sustainability needs, but economic realities may require altering which indicators are considered. For instance, an owner may be willing to

[9] A conceded indicator is one for which an alternative will fail to meet sustainable performance, due to lack of technology, factors that are beyond the realm of control or those that are a necessary byproduct of the nature of the item in question, An example being that current nuclear plants will produce nuclear waste. Conceded indicators must be noted for analysis purposes.

accept reduced environmental performance for an increase in monetary return. Such adjustments are mostly internal to the organization performing the analysis. Adjustments should not completely remove or trivialize key sustainability indicators, as identified in the selection process. While this methodology concedes that economics should not be omitted, it also must be stated that economics should not be used to justify the removal of an indicator whose impact can be alleviated in a financially viable way. A method to achieve an appropriate balance for a selected indicator set is proposed in subsequent sections.

5.1.3 Define Sustainability Requirements for a Project or Product

Each indicator should be tied to a larger sustainability goal. Defining the goal explicitly ensures that the nature of the indicator is fully understood. It also helps to determine if the indicator is necessary and what data is required for appropriate measurement. Considering CO_2 emissions as an example, proposed standards and requirements are designed to alleviate climate change. Therefore, the underlying goal of the CO_2 indicator quantifies how efficient the item is at alleviating greenhouse gas impacts, and thus requires data on CO_2 emissions. Examining governmental regulations on sustainability, voluntary standards such as Leadership in Energy & Environmental Design (LEED) (USGBC 2014) or internally specified sustainability targets can provide guidance in selecting and defining targets.

5.1.4 Collect Data

Data collection follows the selection of indicators and should be driven by the specific needs in the analysis phase. Specifically, the step outlined in probability synthesis requires the following information, as listed in Table 0.1:

1. Target distributions

2. Indicator distributions

3. Correlations among indicators

4. Consequence functions for valuating indicator levels

5. Base values related to implementing an alternative being considered (the cost of an item through its lifecycle, as designed, including any sustainability enhancing features)

6. Revenue[10] the item is expected to generate

[10] Revenue is any income derived from the item, unrelated to the consequences of any sustainability improvement.

Table 0.1. Data collection outline

Data Collection Goal	Needs	Source Types
Probability distributions for converting indicator levels to probabilities of achieving a future need	1. Indicator target values 2. Forecast models 3. Probability target values in meeting future needs 4. An allocation of an indicator level to specific sectors	1. Scientific reports and assessments; i.e. NCA 2. Regulations defining target values 3. Expert opinion elicitation
Probability distributions for the expected indicator levels of a structure through all applicable lifecycle phases	1. Probability distributions for likelihood of obtaining an indicator level	1. Environmental impact studies 2. Economic impact studies 3. Social impact studies 4. Expert opinion elicitation 5. Public forums 6. Lifecycle assessments
Dependencies among indicators	1. Dependencies between indicators	1. Statistical analyses 2. Expert opinion elicitation
Consequence functions	1. Form of consequence value functions 2. Values for consequence function	1. Internal revenue reports 2. Expert opinion elicitation
Utility functions	1. Form of utility functions 2. Multi-attribute utility functional form for consequence and need probability 3. Pertinent values for utility functions	1. Lotteries conducted by decision maker

Table 0.1 outlines the purpose of each data collection requirement and thus the scope of the data collection. Required data for target probability distributions could come from available research, regulations or voluntary standards, while the data for indicator distributions could come from measured values for an existing item or predicted indicator values for a proposed item. Viable data for many indicators, especially in the social pillar, are lacking at present. In the absence of specific data or actual values, typical values may be substituted. For any requirements that cannot be met through data collection, future

efforts should be made to establish a basis for the same. Expert opinion elicitation can bridge many remaining data gaps.

Additional data collection covers economic factors and considerations. Requirements include the set of alternatives being considered, as well as the associated anticipated costs, revenue and consequences. The time value of money (TVM) should be included if pertinent. Each alternative will generate its own set of indicator distributions based on its nature. This increases the number of indicator distributions that need to be estimated, making appropriate screening of alternatives prior to data collection important.

Valuation requires a function, or probability distribution, that maps indicator levels to economic consequences, thereby defining the data collection needs. Valuation must account for economic consequences, such as incurred fines, lost time or lost profit, but not installation or implementation costs (these are included in the base value). Data collection should include any other financial impacts, direct or indirect. Common economic consequences are extra profit from increased market share, savings on utilities for a structure or loss of market share. Potential data sources are internal accounting information, external regulations, market studies or any typical means of obtaining fiscal data. Expert opinion elicitation can fill any data gaps.

5.1.5 Synthesize Probabilities and Consequence Valuations

Once all necessary data are obtained, probability distributions and relationships among indicators must be characterized. Expert opinion elicitation in conjunction with regulatory values, forecast models and proposed indicator targets aid in building the target distributions. Using available data as guidelines, industry and subject matter experts provide estimates of the probability of an indicator meeting or exceeding a sustainability goal. These estimates can be aggregated and used to generate a target distribution.

Indicators vary in terms of relevance and importance. Treating all indicators as equally important may lead to the unimportant indicators skewing results. Methods to account for the relevance and importance of indicators are based on: (1) ad-hoc weighting within the probability calculations (see Appendix B), or (2) adjusting conversion distributions to account for the relative importance of the indicators. A weighting scheme was developed for this methodology; however, the latter option is the focus herein, as ad-hoc procedures may be undesirable. The appropriate starting point for all target distributions is the unbiased case. Adjusting distributions to account for relevance and importance follows. Shifting distribution parameters affects the relative importance of an indicator. Utilizing subject matter experts from all pertinent backgrounds ensures that indicators are treated in an appropriate manner.

Indicator distributions are constructed identically to target distributions, though sources may vary and indicators may have dependencies. While keeping indicators independent simplifies the simulation procedure, it is not necessary for the proposed methodology to work. Elicitation of correlations provides an estimation, if fully decoupling indicators is infeasible.

Consequence valuation requires the determination of the functional dependence of the consequence on the indicator level. For some indicators, the valuation function may already exist or be easily obtainable; for example, in a situation where regulations enforce a fine if a set performance requirement leads to a known negative consequence valuation. Product specifications or building codes may further alter the consequence function.

Expert opinion elicitation or empirical derivation of functions can generate any missing values when no valuation function exists a priori. Consequence functions may include uncertainty if required.

5.1.6 Simulate

Traditional simulation techniques can use the derived distributions from the previous section. Monte Carlo simulation is commonly employed, due to its simplicity. Simulated values must incorporate any correlations defined in the probability synthesis step. The nature of the simulation also prevents correlation between the indicator and target distributions.

Using correlations as described assumes a linear relationship between variables. The linearity assumption may not be true, but determining the joint distribution may be difficult if not impossible. In such cases, simple linear correlation assumptions may be the best approximation available, despite potential inaccuracies. Copulas may also be used if a justifiable dependence structure is available.

The simulation procedure proceeds according to phases and pillars. For the purposes of this methodology, a *phase-pillar* is defined as a collection of indicators that are within the same lifecycle phase and sustainability pillar. Figure 0-2 outlines the simulation process for the case where the individual indicators within a construction phase and pillar are assumed independent.

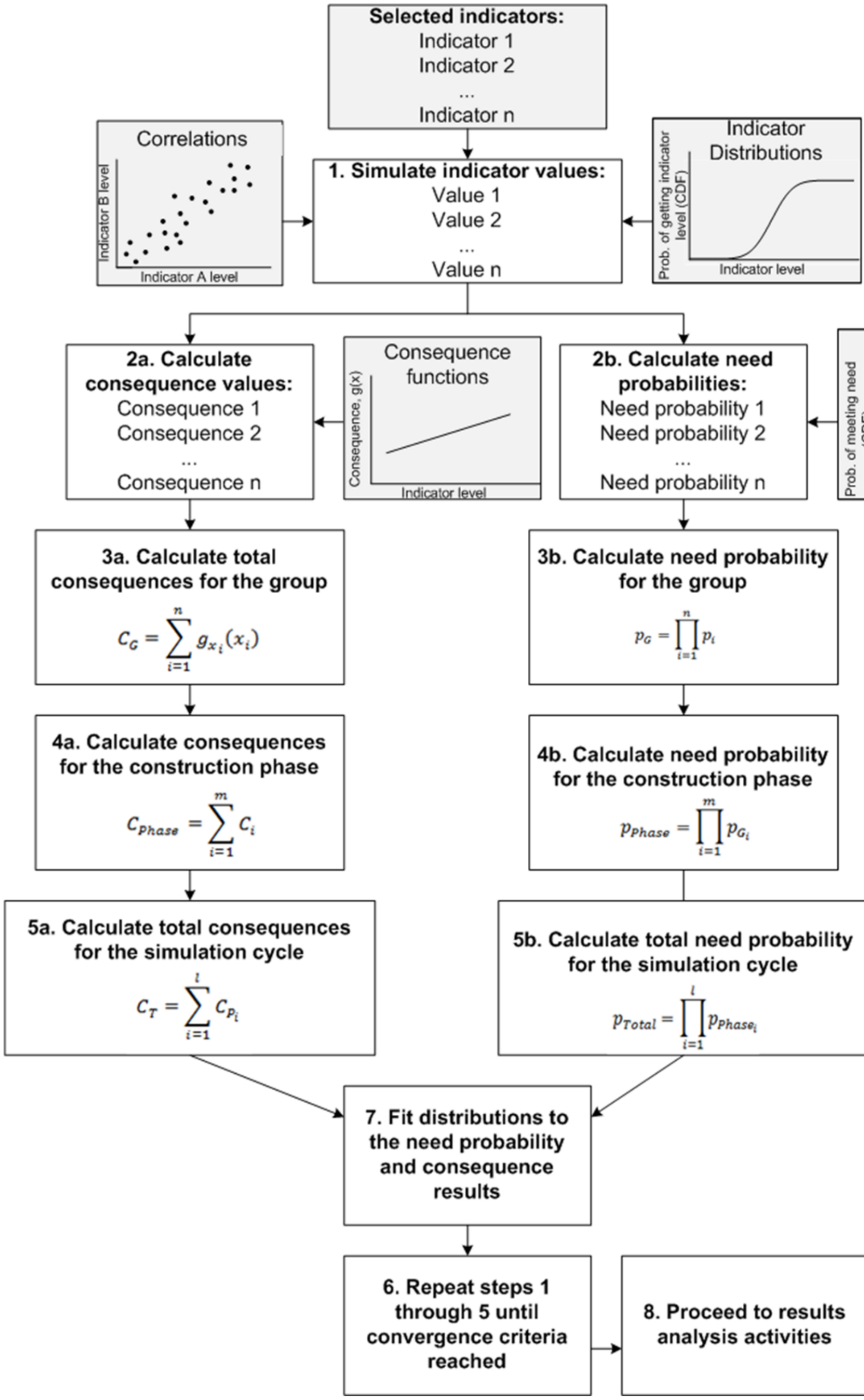


Figure 0-2. Simulation process. Grey – Inputs

Equation 5-1 gives the general formula for a phase-pillar:

$$p_G = \prod_{i=1}^{n} p_i \tag{5-1}$$

where

$n = number\ of\ indicators\ in\ the\ phase - pillar$

$p_G = probability\ of\ meeting\ group\ sustainability\ needs$

$p_i = probability\ of\ the\ i^{th}\ indicator\ meeting\ sustainability\ needs$

To determine the probability of meeting all three pillars within a lifecycle phase, a similar approach applies:

$$p_{Phase} = \prod_{i=1}^{m} p_{G_i} \tag{5-2}$$

where

$m = number\ of\ pillars\ considered$

$p_{Phase} = probability\ of\ meeting\ phase\ sustainability\ needs$

An optimization process can target the most detrimental phases of an item using appropriate results. The final need probability follows as:

$$p_{Total} = \prod_{i=1}^{l} p_{Phase_i} = \prod_{i=1}^{k} p_i \tag{5-3}$$

where

$l = number\ of\ phases$

$k = number\ of\ indicators$

$p_{Total} = probability\ of\ meeting\ all\ sustainability\ needs$

Using Eq. 5-1 through 5-3, each simulation cycle produces a set of probabilities for meeting sustainability needs in each phase-pillar, phase and in total. Alternatively, Eq. 5-4 and 5-5 provide need probabilities within the pillars first, followed by a total. The total need probability can be calculated outright without intermediate results. However, there is substantial benefit in calculating the intermediate results.

$$p_{Pillar} = \prod_{i=1}^{j} p_{G_i} \tag{5-4}$$

where

$j = number\ of\ phases$

$p_{Pillar} = probability\ of\ meeting\ all\ sustainability\ needs\ in\ a\ pillar$

$$p_{Total} = \prod_{i=1}^{k} p_{Pillar_i} \tag{5-5}$$

where

$k = number\ of\ pillars\ being\ considered$

Distributions are fitted to all desired result levels. Traditional statistical analyses follow as necessary. Simulation cycles must also incorporate the economics of sustainability. After randomly generating an indicator level (x_i) according to its distribution, the valuation function (g_{x_i}) from the synthesis step calculates the economic consequences (C_i).

Equations 5-6 through 5-8 derive the total consequence at the simulation iteration level. Consequences can be either positive or negative, with positive consequences representing a beneficial value and negative consequences representing a detrimental value. Consequences can also be probabilistically described, similar to the need probability.

$$C_i = g_{x_i}(x_i) \tag{5-6}$$

$$C_{Iter} = \sum_{i=1}^{n} g_{x_i}(x_i) \tag{5-7}$$

$$C_{Agg} = C_{Iter} + B \tag{5-8}$$

where

$B = the\ cost\ of\ a\ structure\ or\ product\ through\ its\ lifecycle, as\ designed,$

$including\ any\ sustainability\ enhancing\ features$

$C_{Agg} = aggregated\ consequences\ for\ an\ iteration, including\ base\ consequence$

$C_i = consequence\ for\ the\ i^{th}\ indicator$

$C_{Iter} = aggregated\ consequences\ within\ an\ iteration$

$g_{x_i}(x_i) = valuation\ function\ for\ the\ i^{th}\ indicator$

$n = number\ of\ indicators$

The time value of money (TVM) calculations can only be utilized after completing the preceding calculations. The aggregated consequences constitute the total cash flow for a phase. A uniform distribution is a common subjective assumption, and the net present value is defined by Eq. 5-9 to 5-12, assuming a constant annuity and discount rate (*r*).

$$C_{Phase_j} = \sum_{i=1}^{m} g_{x_i}(x_i) \tag{5-9}$$

$$C_{Phase_D} = \frac{C_{Phase}}{t_{Phase}} \tag{5-10}$$

$$C_{Phase_{TVM}} = \frac{C_{Phase_D}}{r}\left[1 - \frac{1}{(1+r)^{t_{Phase}}}\right] \tag{5-11}$$

$$C_T = \sum_{i=1}^{l} C_{Phase_{TVM_i}} \tag{5-12}$$

$C_{Phase_D} = consequence\ of\ the\ phase\ distributed\ over\ phase\ length$

$C_{Phase_j} = consequence\ aggregation\ for\ the\ j^{th}\ phase$

$C_{Phase_{TVM}} = present\ value\ of\ cash\ flow\ from\ phase$

$C_T = total\ net\ present\ value\ of\ consequences$

$m = number\ of\ indicators\ in\ phase\ j$

$l = number\ of\ phases$

$r = discount\ rate\ for\ phase$

$t_{Phase} = length\ of\ phase$

Similar processes can be used based on the net future value or internal rate of return. Ayyub and Klir (2006) presented a discussion on the use of the time value of money calculations in an engineering and risk framework.

5.1.8 Fit Probability Distributions to Simulation Results

In fitting probability distributions to simulation results, the axioms of probability must be preserved. The distribution of need probability must be lower-bounded at zero and upper-bounded at one. Statistical goodness-of-fit methods offer a sound basis for selecting probability distributions. Consequence distributions are not necessarily bounded and may be discontinuous depending on the consequence functions from which it was derived.

5.1.9 Create a Joint Distribution of Consequence and Need Probability

The means for creating the joint distribution of consequence and need probability varies based on the nature of the relationship between the two. A joint cumulative probability distribution (JCDF), $F_{C,P}(c,p)$, offers a generalized means for this purpose. If c and p are independent, the joint probability density function (JPDF) is:

$$f_{P,C}(p,c) = f_P(p) f_C(c) \tag{5-14}$$

If dependency exists, then it must be incorporated into the joint probability distribution. There are multiple means for handling dependency, including the use of copulas (Rüschendorf 2013). A copula is a dependence structure that describes the dependencies between the marginal distributions, called marginals, of the joint distribution. It decouples the effects of the marginal distributions from the dependencies inherent in the joint distribution (Rüschendorf 2013). Appropriate estimation of a copula is a difficult task for larger systems and can cause discrepancies if estimated poorly.

A second solution uses interval calculations to bound the cumulative probability distribution (CDF) between an upper and lower distribution. Any CDF between the upper and lower bounds is possible resulting in interval probabilities. The likelihood of obtaining a particular need probability is between the upper and lower bounds of the interval probability. The process is as follows:

1. Divide the underlying random variable for each distribution into intervals for computational convenience and calculate the probability associated with each interval.
2. Set up a DSS, a table containing all possible combinations of being in both a need probability and consequence interval (an "interval box").
3. Use interval probabilities to calculate bounds in the DSS (Ayyub and Klir 2006).
    a. For the case of independence, a generally invalid, but illustrative, assumption is:

$$Prob(P \cap C) = Prob(P)Prob(C) \tag{5-15}$$

    b. For unknown dependence:

$$Prob(P \cap C) = \left[\max(0, Prob(P) + Prob(C) - 1), \min\left(Prob(P), Prob(C)\right)\right] \tag{5-16}$$

c. For unknown positive dependence (i.e. consequence values increase as the need probability increases):

$$Prob(P \cap C) = \left[Prob(P)Prob(C), \min\left(Prob(P), Prob(C)\right)\right] \tag{5-17}$$

d. For unknown negative dependence (i.e. consequence values decrease as the need probability increases):

$$Prob(P \cap C) = [\max(0, Prob(P) + Prob(C) - 1), Prob(P)Prob(C)] \tag{5-18}$$

e. A heat map may be created to visualize the results.

Using DSS involves calculating probabilities over the partitioned domain. The upper bound of the interval is the sum of all probabilities in intervals that intersect with the value of interest. The lower bound is the sum of all probabilities in intervals that are entirely within the value of interest (Ayyub and Klir 2006). Table 0.2 presents a generic layout of a DSS calculated for a joint distribution. *Prob*, when used in an interval box, represents the probability of being in that interval box.

Table 0.2. Dempster-Shafer structure of joint distribution

	$P = [P_{L1},P_{U1}]$	$P = [P_{L1},P_{U1}]$	...	$P = [P_{Ln},P_{Un}]$
$C = [C_{L1},C_{U1}]$	$Prob_1$	$Prob_2$	...	$Prob_n$
$C = [C_{L2},C_{U2}]$	$Prob_{n+1}$	$Prob_{n+2}$	...	$Prob_{2n}$
...	...	...	...	...
$C = [C_{Lm},C_{Um}]$	$Prob_{(m-1)n+1}$	$Prob_{(m-1)n+2}$	...	$Prob_{mn}$

A subscript "*L*" represents a lower-bound value and a subscript "*U*" represents an upper-bound value. The DSS is an "*m*" by "*n*" matrix with each cell representing a bounded region of need probability and consequence. The "*Prob*" values represent the probability of being within a bounded region calculated using the appropriate case for Eq. 5-15 to Eq. 5-18, based on the assumed dependence.

Dempster-Shafer structures are typically not preferred, due to the associated complexity. While unwieldy and impractical, DSS provides a useful illustration of the nature of uncertainty in these calculations and the importance of its consideration.

5.1.10 Perform Analysis

Decision situations dictate that subsequent analyses are needed, such as benefit-cost analysis of an item for comparison amongst a set of alternatives, directional cosines, sensitivity analysis or hypothesis testing and confidence bounds.

Of interest herein is benefit-cost analysis. Estimating the uncertainties in both consequences and need probabilities is necessary to calculate the probabilities of not realizing the benefits.

Both investment and benefit can be represented by a deterministic value or a probability distribution. Consequence functions map indicator values or need probabilities to the anticipated loss or gain that would result from achieving particular sustainability levels. By defining the terms as above, it becomes possible to treat each separately, and simulate individually, in the proposed methodology. A clear division of monetary flows prevents double counting and maintains independence between revenue and consequence values, which is important for the subsequent analysis.

Equation 5-6 must be altered to the following form for the analysis:

$$BC_{Iter} = \frac{R + C_{Iter}}{B} \tag{5-19}$$

where

$BC_{Iter} = Benefit\ cost\ ratio\ for\ the\ iteration$

$R = Revenue$

Revenue and base values should be simulated if probabilistic in nature. Using benefit-cost while simulating base value and revenue could produce a different dependence with need probability than using consequence values alone. If there is substantial variability in revenue or base value, it may become impossible to determine any strong positive or negative dependence between need probability and benefit-cost ratio. The DSS can model unknown dependence, but if the relationship between base value and revenue dominates the benefit-cost ratio, any consequences may be insignificant in comparison. A scatterplot or statistical test can be created to determine dependence if necessary. The proposed methodology requires probability distributions to be fit to the benefit-cost results, based on the resulting dependence assumptions. Benefit-cost comparisons use the resulting distributions.

Comparing alternatives proceeds through interval calculations. A benefit-cost ratio calculated for the mean or a set percentile is the criterion for comparison. The joint distribution determines the conditional distribution of the need probability associated with the selected criterion, referred to as a “need profile” herein. In lieu of a set percentile value, a minimum benefit-cost ratio can be taken as the criterion and the corresponding need profile determined for it. The need profile simplifies to a single value for comparison by finding the mean of its distribution. Under dependence assumptions, the best that can be obtained is a bounding of the need profile.

To illustrate how a selection criterion can be determined from a DSS, consider the median interval of the need profile. The median interval refers to an interval whose lower bound represents the first instance where the probability bound sum contains 0.5, and whose upper bound represents the last instance where the probability bound sum contains 0.5. In a simplified manner, it is the consequence bound for which all possible instances of the median value are contained. Either the mean or a set percentage of the resulting interval is useful for reducing the most likely interval down to a single benefit-cost value for decision making. Benefit-cost ratios are compared to alternatives and a selection is made based on the largest ratio.

If using a minimum benefit-cost ratio, the first step is to determine its corresponding probability. Interval calculations are performed on the joint DSS to obtain a probability interval. The lower bound probability interval is the interval sum of all interval boxes whose upper bound of the benefit-cost ratio is less than the benefit-cost ratio of interest, while the upper bound interval includes the sum of all interval boxes whose lower bound is less than the benefit-cost of interest (Ayyub 2014). The preferred alternative is the structure or product with the highest probability of exceeding the minimum benefit-cost ratio.

Pre-simulation elicitation and analysis are required to define a desired need profile or need probability for comparison. The benefit-cost ratio for each structure is compared, and whether it meets the requirements for sustainability based on the need profile properties is also determined. If the sustainability requirements are met for all alternatives then the comparison is based on the benefit-cost ratio alone, otherwise options not meeting sustainability requirements are screened out for either rejection or redesign.

Basic uncertainty is inherently quantified for the results. The width of the probability interval indicates how uncertain the value of interest is. A narrower bound indicates less uncertainty, while a wider bound indicates more. It is theoretically possible to find uncertainty bounds for the upper and lower bounds themselves, although it is not as beneficial as the point estimates, because the uncertainty is measured in bounds of bounds, and loses any intuitive meaning. Other basic statistical tests remain available, though may require alteration to be meaningful under DSS.

5.1.11 Create a Joint Distribution for Multiple Items

In the event of quantifying sustainability for multiple items in a portfolio, the procedure must be adapted. Indicator selection proceeds independently for each item in the portfolio. This may result in multiple subsets of indicators with some overlapping. To facilitate analysis, a new indicator set should be constructed including all indicators from the separate items. For instance, consider two structures, S_1 and S_2, with the following indicator subsets, s_1 and s_2, respectively:

$$s_1 = \{Bulk\ CO_2\ emissions, Economic\ impact_1, Accident\ rate\}$$

$$s_2 = \{Untreated\ runoff\ (TSS), Economic\ impact_2, Health\ impacts\}$$

A new indicator set, *s*, is constructed for the aggregated measure, *"s_{agg}"*.

$$s_{agg} = \begin{matrix} \{Bulk\ CO_2\ emissions, Economic\ impact_1, Accident\ rate, \ldots \\ Untreated\ runoff\ (TSS), Economic\ impact_2, Health\ impacts\} \end{matrix}$$

The simulation process assigns a probability of one to any indicator not applicable to an item, assuming that if the indicator is not selected for a structure, it must either meet that specified need implicitly or be of too little importance for inclusion, otherwise it will be conceded. Simulating variables that have need probability set to one will result in the same need probability as not simulating those variables at all. Omitting the variables reduces the data collection required as well as the run time of the simulation.

$$P_{s1}(Meeting\ untreated\ runoff) = 1$$

$$P_{s1}(Health\ impacts) = 1$$

$$P_{s2}(Meeting\ bulk\ C0_2\ emissions) = 1$$

$$P_{s2}(Meeting\ accident\ rate) = 1$$

The simulation procedure applies for any indicators that refer to only one item in the portfolio. For indicators that affect multiple items, the procedure is adjusted. Indicator values must be simulated separately for each structure, based on the respective indicator distributions. If the buildings are of a similar type, are in a similar area, or have other shared aspects, the indicator distributions will most likely be dependent. These dependencies should be estimated for the simulation procedure.

Differences in use, location or other external factors create a different set of requirements for a building. As such, each building may possess distinct target distributions, even if the distributions are for the same indicator. Once all preliminary quantities are established, the previously defined simulation procedure is followed to obtain the results for individual items and the entire portfolio.

In some situations, a full simulation of all buildings is not possible or desired. The PDFs of the final need profile may be the only available information. The joint distribution of the portfolio takes the distribution of one item in it as a marginal. Going from marginal distributions to the joint distribution presents a challenge, as non-uniqueness is present unless independence exists, in which case:

$$f_{P_{Port},C_{Port}}(p_{Tot}, c_{Tot}) = f_{P_1}(p_1) f_{C_1}(c_1) f_{P_2}(p_2) f_{C_2}(c_2) \dots f_{P_n}(p_n) f_{C_n}(c_n) \tag{5-20}$$

$$C_{Port} = random\ variable\ of\ consequence\ for\ entire\ portfolio$$

$$P_{Port} = random\ variable\ of\ need\ probability\ for\ entire\ portfolio$$

To get the probability of the portfolio need probability to be less than or equal to a particular value, a convolution integral needs to be taken with the condition:

$$p_{Port} \leq p_1 p_2 \dots p_n$$

$$p_{Port} = total\ need\ probability\ for\ portfolio$$

for the consequence values:

$$c_{port} \leq c_1 + c_2 + \cdots + c_n$$

$$c_{port} = total\ consequence\ for\ portfolio$$

If dependence exists, bounded probabilities are prescribed as follows:

1. Determine the joint distribution, $F_{C,P}(c,p)$, for each item using the process described. Dempster-Schafer structures define the joint distributions.
2. Calculate new bounds for need probability and consequence values for the joint distribution of the portfolio using interval arithmetic.

$$[a_L, a_U] + [b_L, b_U] = [a_L + b_L, a_U + b_U] \tag{5-21}$$

$$[a_L, a_U] * [b_L, b_U] = [a_L * b_L, a_U * b_U] \tag{5-22}$$

3. Set up a new Dempster-Schafer structure for the portfolio joint distribution and calculate the probabilities of being in new intervals determined using Eq. 6-15 to 6-18.
4. Applying a dependence assumption may cause the intervals to become too wide to be meaningful. In such cases it may be beneficial to take the average of the interval or set up a correlation relationship as follows:
    a. Establish a relationship between a qualitative correlation and the percentage of the probability range. For example, two structures may be expected to have low correlation. An assumed value for such an anticipated correlation could be 0.30 percent of the range.
    b. Visualize the portfolio using a heat map.

Interval calculations on the DSS determine the consequence and need probabilities (Ayyub and Klir 2006).

The benefit-cost analysis described herein is implementable across an entire portfolio if desired. The analysis produces the expected base value and revenue for the entire portfolio. Equation 5-23 calculates the portion of the portfolio benefit-cost ratio for each item.

$$BC_{partial_i} = \frac{C_{iter_i} + R_{port}}{B_{port}} \tag{5-23}$$

where

$B_{port} = base\ value\ for\ the\ entire\ portfolio$

$BC_{partial_i} = portion\ of\ benefit - cost\ ratio\ from\ i^{th}\ item\ in\ portfolio$

$R_{port} = revenue\ for\ the\ entire\ portfolio$

Convolution integrals using the sum of the partial benefit-cost ratios determine the probability of the total benefit-cost ratio being less than a particular value. The remaining process is identical to the convolution integral process previously described.

5.1.12 Revisit the Spider Diagram

The spider diagram retains utility in the proposed methodology. The diagram now requires the plotting of the PDF for each result of interest (construction phase, pillar, etc.). Plots of alternatives, or a set of ideal characteristics, can be simultaneously overlaid for comparison, as in Figure 0-3.

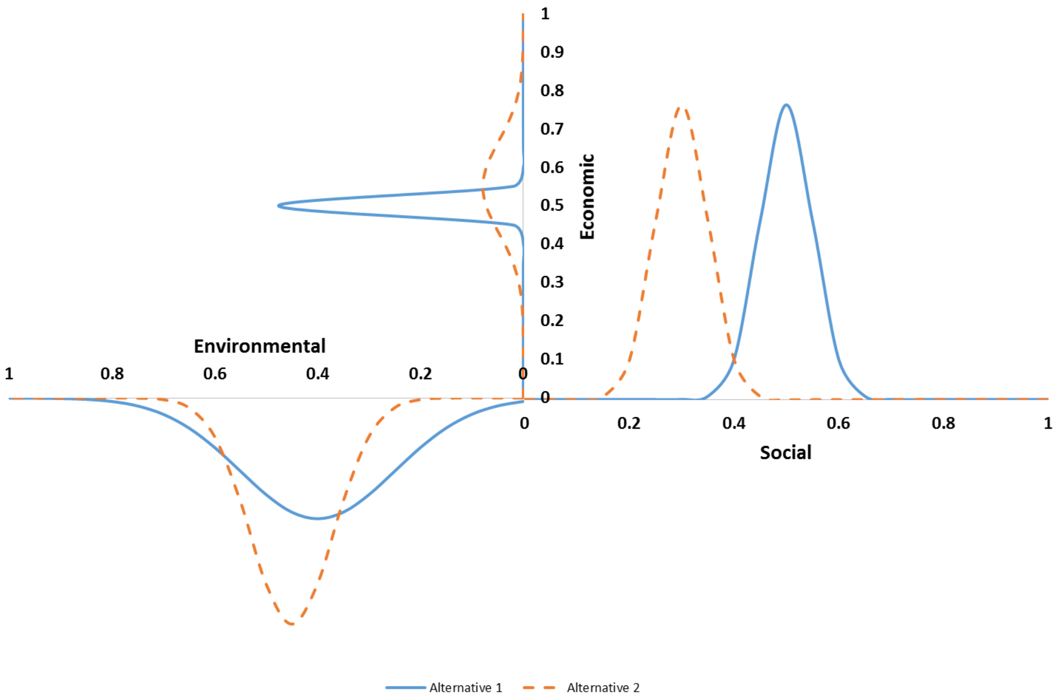


Figure 0-3. Spider diagram incorporating uncertainty

Making decisions under uncertainty becomes difficult, if not impossible, due to the distributed nature of the results, although in some cases, a distribution may be clearly favored over another, as in Figure 0-4.

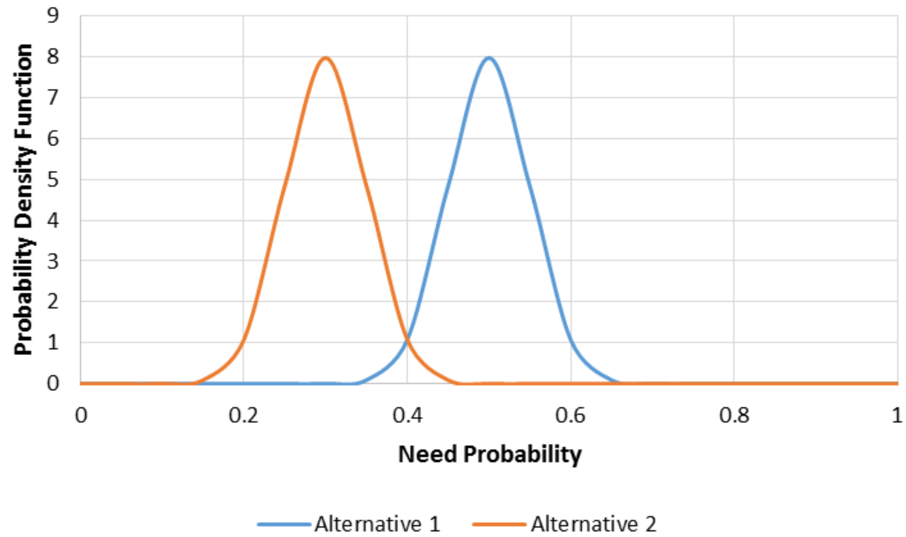


Figure 0-4. Case of clear dominance (Alternative 1 over Alternative 2)

If the plot looks more like Figure 0-5 however, the preferred option may be less obvious.

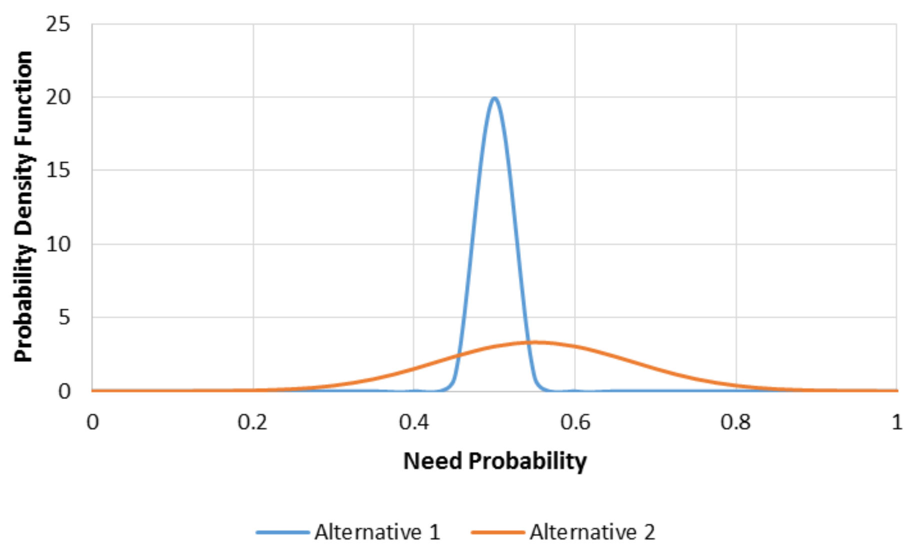


Figure 0-5. Case of unclear dominance

To use the spider diagram to make comparisons under uncertainty, the probability that one alternative has better characteristics than another is determined. Consider the two normal distributions in Figure 0-5 defined as:

$$P_1 \sim N(0.5, 0.02)$$

$$P_1 \sim N(0.55, 0.12)$$

In this case, the better distribution is harder to identify. The mean is a potential comparison criterion, in which case Alternative 2 would be chosen, however its higher standard deviation indicates more uncertainty in the value. A better method of comparison is to determine the probability that one alternative has a higher aggregated need probability than the other. For the case of two normal distributions, the already formulated stress-strength interference model, Equation 5-24, can be used.

$$Prob(Alternative\ 1 > Alternative\ 2) = \Phi\left[\frac{\mu_1 - \mu_2}{\sqrt{{\sigma_1}^2 + {\sigma_2}^2}}\right] \tag{5-24}$$

This formula produces:

$$Prob(Alternative\ 1 > Alternative\ 2) = \Phi\left[\frac{0.5 - 0.55}{\sqrt{0.02^2 + 0.12^2}}\right] = 0.341$$

Therefore, the probability that Alternative 1 will have a better aggregated need probability than Alternative 2 is 0.36. Alternative 2 should thus be chosen.

The distributions representing need probabilities are limited to the interval [0,1] to maintain consistency with the axioms of probability. Normal distributions mimic such conditions if the combination of the mean and standard deviation result in the tails "dying off" in such a way that rounding the CDF at the bounds results in the appropriate values. While tempting, using a truncated normal distribution for the stress-strength interference equation is risky. If too much of the distribution is truncated, the assumption of normality can no longer be considered valid. Consider Figure 0-6 showing a distribution with a mean of 0.95 and a standard deviation of 0.1.

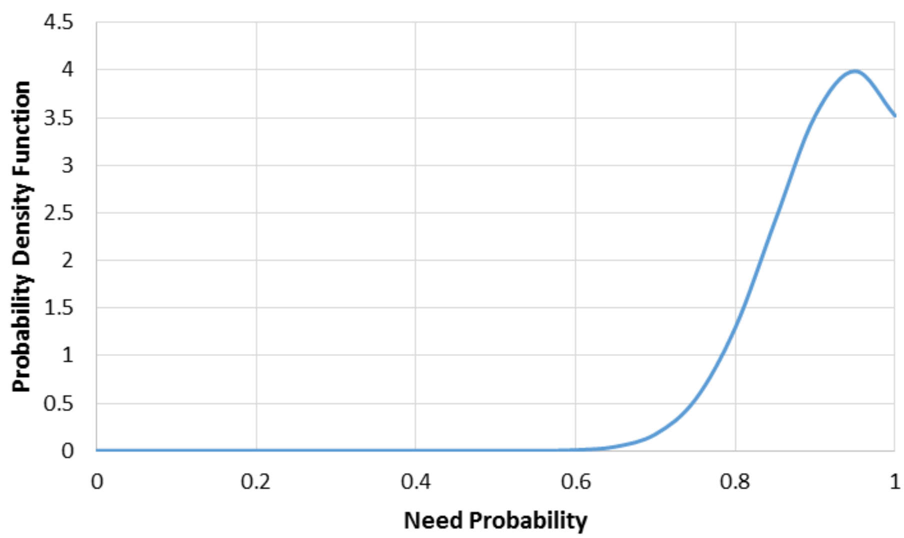


Figure 0-6. Truncated distribution within viable need probability range

Truncating the distribution in Figure 0-6 at one removes 21% of the area under the curve, making the truncated distribution between zero and one non-normal. When simple formulas are unavailable, a simulation technique may be implemented.

5.1.13 Relate the Proposed Methodology to Current Methods

The proposed methodology shares some similarities with existing methods and can easily be adapted. The primary comparison is to that of traditional LCA modeling. Both are meant to be a "cradle-to-grave" analysis that is heavily reliant on data. In fact, the environmental flows and the underlying data used to generate the analyses could be used as inputs into the proposed methodology. Both also utilize expert opinion elicitation in the absence of data and attempt to adjust the relative importance of certain indicators. In LCA this is done by weighting the inputs in the environmental impact score, while the proposed methodology focuses on the adjustment of the target distributions, although a weighting scheme is also available. The proposed methodology shares a similarity with the PSI methodology in that both convert results to probabilities based on phase and/or pillar.

The proposed methodology could be adapted to the LCA methodology by removing the target distributions. The indicator distributions would be generated from the inventory data and used to simulate the environmental flows that go into the environmental impact score. Multiple simulations of the LCA procedure could then generate a distribution of environmental impact scores for analysis.

Where the proposed methodology differs is in the use of need probability. The need probability is an explicit attempt to relate the indicator level to a sustainability goal. Uncertainty is a requirement for the proposed methodology, not an optional calculation.

As previously noted, the lack of uncertainty in most methods is a result of the difficulties in data collection, however there needs to be an effort to understand the methods as data collection and sustainability practices move forward. The proposed methodology also produces a universal metric, a probability, instead of a score or index that has little meaning outside the specific method used to generate it.[11]

The proposed methodology also explicitly considers the economic impacts of sustainability as a feature. Linking sustainability performance to the potential gains and losses incurred allows practitioners to better understand the full impact of any measures taken. In short, it does no good to operate a business that is highly sustainable, but fails quickly due to a shortsighted view of the fiscal realities of an operation.

[11] That is not to say LCA or any other index is not useful, just that their results are often narrow to their specific context.

Chapter 6: Illustrative Dempster-Shafer Examples

This section contains material from the paper *Sustainability Quantification and Valuation. II: Probabilistic framework and metrics for sustainable construction* that has been accepted for publication in the ASCE-ASME Journal for Risk and Uncertainty in Engineering Systems Part A: Civil Engineering Vol 3(3) (Webb and Ayyub 2016b).

Two basic examples and one comprehensive example were developed to illustrate the DSS method. The steps follow those outlined in Chapter 6.1. Examples 1 and 2 illustrate the process in a way that is easy to follow and replicate. The values used in these basic examples are not based on any literature, engineering judgement or actual structures. The comprehensive example is a more rigorous test of the methodology's applicability and its ease of implementation. While an attempt was made to create a semi-realistic structure using typical values, in many cases values had to be assumed.

Each of the examples that follows is meant to be illustrative of the process, and the results should not be considered meaningful in numeric context. This is the result of assumptions made out of necessity, or in order to simplify the analysis to aid in the illustrative nature of the examples. In general, typical values were obtained from literature where applicable for the indicator distributions. Target distributions were generated from published works where possible and assumed when no values could be found. Uncertainty was added under assumed distribution types and with assumed coefficients of variation and correlations. Consequence functions were assumed linear. Because of the illustrative nature of the examples, no sensitivity analysis was performed on the inputs.

6.1 Illustrative Basic Example 1 Results Following the Proposed Process

These basic examples considered only two phases of all three pillars to simplify the calculations. One indicator was selected for each pillar: CO_2 emissions for the environmental pillar, jobless rate for the economic pillar and accident rate for the social pillar. The goals for each indicator were as follows:

- CO_2 emissions: reduce emissions to a level that will decrease the carbon footprint of the building, lowering the global warming impact
- Jobless rate: ensure economic sustainability by providing sufficient employment
- Accident rate: ensure workplace safety by preventing serious accidents

The basic examples were simplified for computational convenience and both omitted data collection. Functional forms for consequence functions, conversion distributions and indicator distributions were assumed. All indicators were treated as independent. Table 10 summarizes the assumed relationships for all indicators.

The value of the alternative, excluding consequence numeration, was assumed to be 3.0. Revenue from the alternative was 2.5 (assumed) in the same notional monetary units as the consequences from Table 0.1.

Table 0.1. Definition of variables for basic example 1

Indicator	Group	Type	Functional form	Mean†	Standard Deviation
CO_2 emissions	Environment-Construction	Conversion distribution	Lognormal	10.7	7
		Indicator distribution	Lognormal	11	1.2
	Environment-Operation	Conversion distribution	Lognormal	21.2	11
		Indicator distribution	Lognormal	22.3	3.3
	Environment-All phases	Consequence function	$C = \exp\left(\mu_y + \sigma_y \Phi^{-1}(p)\right)$ ††	0.8	0.2
Jobless rate	Economic-Construction	Conversion distribution	Lognormal	3.2%	0.6%
		Indicator distribution	Lognormal	3.6%	2%
	Economic-Operation	Conversion distribution	Lognormal	2.7%	0.3%
		Indicator distribution	Lognormal	2.8%	1.9%
	Economic-All phases	Consequence function	$C = \exp\left(\mu_y + \sigma_y \Phi^{-1}(p)\right)$	0.5	0.1
Accident rate	Social-Construction	Conversion distribution	Lognormal	1/30	1/5
		Indicator distribution	Lognormal	1/25	1/35
	Social-Operation	Conversion distribution	Lognormal	1/65	1/10
		Indicator distribution	Lognormal	1/55	1/60
	Social-All phases	Consequence function	$C = \exp\left(\mu_y + \sigma_y \Phi^{-1}(p)\right)$	1.0	0.3
Utility functions	Total consequence	Utility function	$U_C(c) = 1 - \exp\left(-\frac{c}{0.6}\right)$	-	-
	Total need probability	Utility function	$U_P(p) = 1 - \exp\left(-\frac{p}{0.4}\right)$	-	-
	Combined total consequence and need probability	Multi-attribute utility function	$U_{C,P}(c,p) = 0.3U_P(p) + 0.6U_C(c) + 0.1U_C(c)U_P(p)$	-	-

The simulation procedure previously described was followed using 5000 simulations. Output distributions from the methodology are given in Eq. 6-1 and 6-2. The means and standard deviations in the output distributions were in the non-transformed space, i.e. representing the real mean and standard deviation, not the lognormal parameters.

$$P \sim LN(0.000644{,}0.000762) \tag{6-1}$$

$$C \sim LN(3.809{,}0.133) \tag{6-2}$$

$$P \sim LN(\mu_x, \sigma_x) = P\ is\ lognormally\ distributed\ with\ non - transformed\ mean\ \mu_x\ and\ non - transformed\ standard\ deviation\ \sigma_x$$

Consistency with the probability theorems required that the need probability distribution be bounded at zero and one. The fit lognormal distribution is naturally bounded at zero but is unbounded for positive integers. In this instance, truncation at the upper bound was unnecessary, as the precision limit of the software was reached prior to a need probability of one, causing any cumulative distribution function (CDF) value of need probability greater than unity to be rounded to one automatically. Truncation was therefore unnecessary for calculation purposes. In cases where truncation may be necessary, the CDF should be normalized to facilitate proper probabilistic interpretation.

Equations 6-1 and 6-2 provide the marginal distributions for the respective variables. The joint cumulative distribution function (JCDF) of need probability and consequence in lognormal space is given in Eq. 6-3, where μ_{P_y} and μ_{C_y} are the lognormal means of the output need probability and consequence, respectively. Similarly, σ_{P_y} and σ_{C_y} are, respectively, the lognormal standard deviations of the need probability and consequence. The numerical representation of the distribution is given in Eq. 6-4.

$$F_{C,P}(c,p) = \Phi\left(\frac{\ln p - \mu_{P_y}}{\sigma_{P_y}}\right)\Phi\left(\frac{\ln c - \mu_{C_y}}{\sigma_{C_y}}\right) \tag{6-3}$$

$$F_{C,P}(c,p) = \Phi\left(\frac{\ln p + 7.786}{0.936}\right)\Phi\left(\frac{\ln c - 1.337}{0.0349}\right) \tag{6-4}$$

The distribution of the benefit/cost ratio (BCR) is provided as an illustration of a possible analysis following the proposed methodology. The JCDF of need probability and benefit/cost ratio is given in Eq. 6-5, assuming the need probability and consequence are independent.

$$\begin{gathered} F_{BCR,P}(bcr,p) \\ = \Phi\left(\frac{\ln p + 7.786}{0.936}\right)\Phi\left(\frac{\ln bcr - 0.743}{0.0209}\right) \\ bcr = benefit/cost\ ratio \end{gathered} \tag{6-5}$$

The JCDF can be simplified if the mean BCR (2.103) is used as the comparison criteria. The corresponding need profile, defined herein as the need probability distribution for the calculated mean benefit/cost ratio, is given in Eq. 6-6. The mean value of the need profile portion of the JCDF (0.000317) can also be calculated under the assumed BCR condition.

$$F_{P|BCR}(p, bcr \geq 2.103) = 0.493\Phi\left(\frac{\ln p + 7.786}{0.936}\right) \tag{6-6}$$

Benefit/cost ratio was used for an alternative comparison, as well as for checking the viability of a single option. To illustrate such a comparison, the previous calculations were assumed to be for a building denoted Alternative A. A second building, Alternative B, was analyzed under the same methodology and found to have a mean benefit/cost ratio of 3.5 in the non-transformed space. The mean of the need profile for Alternative B was 0.000168. Alternative B would be selected if only the BCR were used, as its value of 3.5 would be greater than Alternative A's value of 2.103. If a further condition was added, such that the mean of the need profile must be at least 0.0002, then Alternative A would have been selected.

Stochastic dominance is another tool used to compare probabilistic options. The need probability distribution, P2, was assumed to illustrate this concept.

$$P_2 \sim LN(0.0005, 0.00008)$$

The probability that Building A had a higher need probability than Building B was found to be 0.428. The stress interference model is only closed-form for a limited number of distributions. For other distribution types, simulation methods can be used. The same comparison as above was conducted using a simulation with 2000 cycles. Under the simulation procedure, the probability that Building A had a higher need probability was 0.431; very close to the exact value of 0.428. Both results indicated that the second building was more likely to have a higher need probability and therefore better sustainability. A similar procedure would be viable for comparing consequences, benefit/cost ratios or any other distributions of interest.

Building A may not be considered isolated, in terms of an analysis. If it is a part of a portfolio of buildings, a portfolio-wide JCDF may be desired. For instance, if a second building, assumed identical but independent to Building A, was under consideration, a JPDF of the two buildings would be defined as follows:

$$f_{C,P}(c_1, c_2, p_1, p_2)$$
$$= \phi\left(\frac{\ln p_1 + 7.786}{0.936}\right)\phi\left(\frac{\ln c_1 - 1.337}{0.0349}\right)\phi\left(\frac{\ln p_2 + 7.786}{0.936}\right)\phi\left(\frac{\ln c_2 - 1.337}{0.0349}\right)$$

A convolution integral was taken to obtain the probability of a specific outcome using the JPDF:

$$P(v,u) = \int_0^{v/p_1} \int_0^{p_1} \int_{-\infty}^{w-c_1} \int_{-\infty}^{c_1} \phi\left(\frac{\ln p_1 + 7.786}{0.936}\right) \phi\left(\frac{\ln c_1 - 1.337}{0.0349}\right) * \phi\left(\frac{\ln p_2 + 7.786}{0.936}\right) \phi\left(\frac{\ln c_2 - 1.337}{0.0349}\right) dc_1\, dc_2 dp_1 dp_2$$

$v\ =\ particular\ product\ of\ need\ probabilities\ for\ the\ entire\ portfolio$

$w\ =\ particular\ sum\ of\ consequences\ for\ the\ entire\ portfolio$

6.2 Illustrative Basic Example 1 Discussion

Basic example 1 illustrated the proposed methodology. Inputs and results were notional in nature, though demonstrative and verifiable. The formulaic process and the use of probability allowed for a meaningful interpretation of results beyond the immediate output. The primary concern in the case of independence was within the initial steps of the procedure. If a large amount of expert elicitation is required, uncertainty or bias in the data collection step will carry through the entire process, which could lead to substantial errors in the final results. This concern is rather uncommon in LCA, as different methodologies or practitioners obtain different results.

6.3 Illustrative Basic Example 2: Illustrative Example Assuming Dependence between Multiple Buildings

Basic example 2 begins at step 7 of the prescribed procedure, as the methods for independence and dependence are identical up to this point. This basic example illustrates the use of DSS to account for dependence. For benefit/cost ratio purposes, a base value of 3.0 and an expected revenue of 3.5 in notional monetary units identical to those used for consequence were assumed.

Let Eq. 23 be the PDF for need probability. Equation 6-7 was assumed for computational purposes and was not based on engineering judgement or previous work.

$$f_P(p) = \frac{1}{B(2,5)} p^1 (1-p)^4 \tag{6-7}$$

Consequences were assumed to follow a lognormal distribution with a lognormal mean and standard deviation of 0.9871 and 0.2231, respectively. Unknown positive dependence was assumed to govern the relationship between need probability and consequence. The joint distribution of the two variables was modeled using a DSS.

Constructing the Dempster-Schafer structure first required the determination of an upper bound for the consequence. Because consequence can theoretically be infinite according to the lognormal distribution, a value was chosen such that its CDF value was approximately one. A consequence value of 4.5 suited this purpose. The marginal distributions for the need probability and consequence were then divided into intervals. Interval size was a function of the desired level of resolution for the results, the nature of the distributions themselves and the need to not be equal. Two intervals were chosen for both need probability and consequence in order to keep the DSS reportable. Following the procedure described previously, the DSS for the joint distribution in Table 0.2 was obtained. *Prob*, when used in a row or column heading of a Dempster-Schafer structure in this paper, indicates the probability of being in the specified interval.

Table 0.2. Dempster-Schafer structure for basic example 2 for the joint structure of consequence and need probability

P / C	P [0,0.5] Prob = 0.890	P [0.5,1.0] Prob = 0.110
C [0,2.25] Prob = 0.215	Prob(c,p) = [0.191,0.215]	Prob(c,p) = [0.024,0.110]
C [2.25,4.5] Prob = 0.775	Prob(c,p) = [0.690,0.775]	Prob(c,p) = [0.085,0.110]

For benefit/cost comparison purposes, the Dempster-Shafer structure in Table 0.2 was recalculated to produce Table 0.3.

Table 0.3. Dempster-Shafer structure for basic example 2 for benefit/cost ratio

P / BCR	P [0,0.5] Prob = 0.890	P [0.5,1.0] Prob = 0.110
BCR [1.167,1.917] Prob = 0.215	Prob(bcr,p) = [0.191,0.215]	Prob(bcr,p) = [0.024,0.110]
BCR [1.197,2.333] Prob = 0.775	Prob(bcr,p) = [0.690,0.775]	Prob(bcr,p) = [0.085,0.110]

The median value provided a fair criterion for comparison. The small number of intervals in this example made obtaining an accurate estimate difficult. A realistic analysis would require more interval boxes for an appropriate resolution. The best estimate of the median was between 1.197 and 4.5. The mean of this range, 2.846, was deemed suitable for single value comparisons.

A need profile was built for the selected median benefit/cost ratio range by creating a new DSS for the median range. The bounds on the need probability intervals were nested and lower bounded at zero. The upper bounds were selected at sufficient increments to accurately model the CDF of the need profile. The bounds were arbitrarily chosen for illustrative purposes. Table 0.4 provides the resulting benefit/cost ratios. The probabilities in Table 0.4 were normalized to one where required to ensure a valid CDF.

Table 0.4. Need profile for median range of benefit/cost ratio for the building in basic example 2

P / BCR	P [0,0.25] Prob = 0.45	P [0,0.5] Prob = 0.89	P [0,0.75] Prob = 0.92	P [0,1.0] Prob = 1.0
BCR [1.197,2.333] Prob= 0.775	Prob(bcr,p) = [0.45,0.58]	Prob(bcr,p) = [0.89,1.0]	Prob(bcr,p) = [0.92,1.0]	Prob(bcr,p) = [1.0,1.0]

Comparison of results when using bounded probabilities is difficult. A practical criterion is a set percentile of the need profile. For instance, the median of the need profile could be used; the range of 0 to 0.25 was used in this example. This range can be further simplified to its average of 0.125. Comparison then proceeded in the same manner as in basic example 1.

To apply the probability bounding procedure to multiple buildings, a building, denoted Building 2, was assumed to have the DSS provided in Table 0.5. Building 1 refers to the building with the DSS given in Table 0.2.

Table 0.5. Dempster-Schafer structure of Building 2 for basic example 2

P / C	P [0,0.4] Prob = 0.890	P [0.4,1.0] Prob = 0.110
C [0,3] Prob = 0.450	Prob(c,p) = [0.400,0.450]	Prob(c,p) = [0.050,0.110]
C [3,6] Prob = 0.550	Prob(c,p) = [0.490,0.550]	Prob(c,p) = [0.061,0.110]

The joint structure of Building 1 and Building 2 had the form provided in Table 0.6.

Table 0.6. Empty Dempster-Schafer structure for the joint distribution of need probability and consequence for Building 1 and Building 2

C \ P	P = [0,0.2]	P = [0,0.5]	P = [0,0.4]	P = [0.2,1.0]
C = [0,5.25]	$Prob_1$	$Prob_2$	$Prob_3$	$Prob_4$
C = [3,8.25]	$Prob_5$	$Prob_6$	$Prob_7$	$Prob_8$
C = [2.25,7.5]	$Prob_9$	$Prob_{10}$	$Prob_{11}$	$Prob_{12}$
C = [5.25,10.5]	$Prob_{13}$	$Prob_{14}$	$Prob_{15}$	$Prob_{16}$

The probability bound of the Xth interval box to be computed was denoted *ProbX*; for instance, Eq. 6-8 calculates the probability for the first probability box, *Prob1* as:

$$Prob_1 = Prob(P_{S1} = [0,0.5] \cap P_{S2} = [0,0.4] \cap C_{S1} = [0,2.25] \cap C_{S2} = [0,3]) \qquad (6\text{-}8)$$

If the two buildings are independent, the bounds are calculated per Eq. 6-9.

$$Prob_{1_{Ind}} = [0.191(0.4), 0.215(0.450)] = [0.0764, 0.0967] \qquad (6\text{-}9)$$

If unknown positive dependence holds, Eq. 6-10 is used.

$$Prob_{1_{Dep}} = \big[[0.191(0.4), min(0.191,0.4)], [0.215(0.45), min(0.215,0.45)]\big] \qquad (6\text{-}10)$$

$$Prob_{1_{Dep}} = \big[[0.0764, 0.191], [0.0967, 0.215]\big]$$

$$\mathrm{Prob}_{1_{\mathrm{Dep}}} = \text{probability for interval box 1 assuming unknown positive dependence}$$

Simplifying to the lowest lower bound and the largest upper bound of the unknown positive dependence case yielded:

$$Prob_{1_{Dep}} = [0.0764, 0.215]$$

The bounds indicated that the probability of achieving a need probability of 0 to 0.2 simultaneously with a consequence value of 0 to 5.25 was between 0.0764 and 0.215. Table 0.7 provides the complete DSS calculated assuming independence between the buildings.

Table 0.7. Calculated Dempster-Schafer structure for Table 0.6 (Columns are numbered for reference in need probability CDF calculations)

P / C	1. P = [0,0.2]	2. P = [0,0.5]	3. P = [0,0.4]	4. P = [0.2,1.0]
C = [0,5.25]	[0.076,0.097]	[0.010,0.024]	[0.001,0.050]	[0.0012,0.0121]
C = [3,8.25]	[0.094,0.118]	[0.011,0.024]	[0.012,0.061]	[0.0014,0.0121]
C = [2.25,7.5]	[0.276,0.349]	[0.035,0.085]	[0.034,0.050]	[0.0043,0.0121]
C = [5.25,10.5]	[0.338,0.426]	[0.042,0.085]	[0.050,0.061]	[0.0052,0.0121]

Utilizing DSS for probability calculations was not as intuitive as in the independent case. To illustrate, consider the probability of achieving a need probability of 0.45 or less was desired. The probability bounds were calculated based on the concept of rough sets (Ayyub and Klir 2006). The lower bound was the sum of the probability bounds for all intervals with upper bounds less than the value of interest. For the assumed value, any need probability interval with an upper bound of less than 0.45 was selected; in this case the first and third columns in Table 0.7. This is expressed mathematically in Eq. 6-11.

$$\underline{Prob}(P \leq 0.45) = \sum_{i=1,3} all\ intervals\ in\ column\ i$$
$$= [min(0.881,1)\,, min(1.212,1)] \tag{6-11}$$

$$\underline{Prob} = \ lower\ bound\ of\ the\ probabilty\ interval$$

The upper bound was the sum of all probability bounds with a range containing the value of interest. Any range that contained 0.45 was included, which were all columns in Table 0.7 in this case, and summed per Eq. 6-12.

$$\overline{Prob}(P \leq 0.45) = \sum_{i=1,2,3,4} all\ intervals\ in\ column\ i$$
$$= [min(1,1)\,, min(1.478,1)] \tag{6-12}$$

$$\overline{Prob} = \ upper\ bound\ of\ the\ probabilty\ interval$$

Note that the sum of the upper bounds was greater than one in both cases. Calculations on probability bounds have this issue when assuming unknown positive dependence or unknown dependence. Since the upper bound must be greater than the probability when assuming independence, a summation of all upper bounds will exceed unity. The use of the “minimum” function in Eq. 6-11 and Eq. 6-12 corrected the apparent violation of the axioms of probability. The final probability bound is given in Eq. 6-13.

$$Prob(P \leq 0.45) = [0.881,1] \tag{6-13}$$

6.4 Illustrative Basic Example 2 Discussion

Similar to basic example 1, the numeric results were verifiable given the input but not representative of a real building. A key observation was that the probability bounds were unlikely to be desirable in practical applications. A single value is typically the goal for design or optimization. The final probability range may need to be simplified further to a single value in practice, either by choosing the average of the range or some set percentile. If justifiable, a separate distribution could be assumed over the probability interval. An example would be a four-parameter beta distribution bounded by the range given in Eq. 6-13. Such a distribution, however, would need to be determined subjectively through expert elicitation.

6.5 Illustrative Comprehensive Example

The comprehensive example illustrates the proposed process more thoroughly using the construction industry as the application domain. A medium-sized office building was selected without a specified location, facilitating the use of data sources across multiple geographical regions. Three phases were considered: construction, operation and decommissioning. Sustainability efforts should begin in the design phase; however, the phases selected are sufficient to display the proposed model's efficacy. The analysis began by examining a single building. Adding a second building to create a portfolio of buildings followed. Dempster-Schafer structures were also introduced in the last section. The time value of money calculations and true building life-cycle cost analysis were omitted. While engineering economic considerations and appropriate costing are vital in practice, the removal here simplifies the demonstration.

6.5.1 Individual Building Calculations

All three sustainability pillars were considered in each of the selected phases. The needs of specific pillars drove the selection of indicators. Environmental indicators focused on typical pollution sources, including CO_2 emissions, resource usage indicators and runoff. Economic indicators focused on economic impacts and property values, while social indicators focused on health impacts and economic equity.

The normalized Theil's T statistic was utilized as a normalized ratio measuring social equity. A value of zero represented complete equality, while one equated to absolute disparity (Haughton and Khandker 2009). Table 0.8 lists the indicators chosen for the comprehensive example and the typical values pulled from the literature.

Table 0.8. Typical building data used for comprehensive example indicators

Phase	Indicator	Value	Source
Construction	Bulk CO_2 emissions	2.6 Mt/yr.	Dept. of Business Innovation and Skill (BIS 2010)
	Recycled materials used	40%	Environmental Protection Agency (EPA 2009a)
	Total suspended solids	6000 lb/acre-yr.	(EPA 2006)
	Percentage waste	20%	(Bossnick and Browers 1996)
	Road closure impacts	10000 vehicle-hours/day	Rounded down from 10192.6 in Federal Highway Administration example (USDOT 2005)
	On-site accident rate	1 per 25 days	Assumed
	Regional health impact	2%	Assumed
	Theil statistic	0.41	Assumed
Operation	Bulk CO_2 emissions	100.7 Mt/yr.	(BIS 2010)
	Energy intensity	79.8 kBtu/ft^2	(ILuvTrees 2009)
	Water usage	15 gal/ft^2	(EPA 2012)
	Total suspended solids	1000 lb/acre-yr.	(EPA 2006)
	Economic impact	0.5%	Assumed
	Property values	0.2%	Assumed
	On-site accident rate	1 per 30 days	Assumed
	Regional health impact	0.05%	Assumed
	Theil statistic	0.35	Assumed
Decommissioning	Bulk CO_2 emissions	1.3 Mt/yr	(BIS 2010)
	Materials recycled	40%	(EPA 2009b)
	Total suspended solids	6000 lb/acre-yr.	(EPA 2006)
	Road closure impacts	10000 Vehicle-hours/day	(USDOT 2005)
	Economic impact	-0.3%	Assumed
	Property values	-0.1%	Assumed
	On-site accident rate	1 per 20 days	Assumed
	Regional health impact	0%	Assumed
	Theil statistic	0.43	Assumed

Table 0.8 should not be considered exhaustive, and any indicators chosen were not necessarily applicable across all buildings. Table 0.9 provides the sustainability goals to which each indicator relates.

Table 0.9. Targets for indicators used for comprehensive example

Pillar	Phase	Indicators	Units
Environment	Construction	Bulk CO_2 emissions	Alleviate climate change impacts
		Recycled materials used	Protect natural resources by not using as many virgin sources
		Total suspended solids	Prevent pollution of waterways due to runoff
		Percent waste	Prevent unnecessary waste
	Operation	Bulk CO_2 emissions	Alleviate climate change impacts
		Energy intensity	Prevent additional emissions due to power plant
		Water usage	Prevent overuse of water resources
		Total suspended solids	Prevent pollution of waterways due to runoff
	Decommissioning	Bulk CO_2 emissions	Alleviate climate change impacts
		Materials recycled	Prevent unnecessary waste
		Total suspended solids	Prevent pollution of waterways due to runoff
Economy	Construction	Road closure impacts	Prevent undue burden on citizens
	Operation	Economic impact	Ensure overall benefit from constructing the building
		Property values	Ensure overall benefit from constructing the building
	Decommissioning	Road closure impacts	Vehicle-hours lost
		Economic impact	Ensure overall benefit from constructing the building
		Property values	Ensure overall benefit from constructing the building
Social	Construction	On-site accident rate	Ensure worker safety
		Regional health impacts	Ensure no adverse health effects on population
		Normalized Theil's T statistic	Ensure all impacts are distributed across social classes equally
	Operation	On-site accident rate	Ensure worker safety
		Regional health impacts	Ensure no adverse health effects on population
		Normalized Theil's T statistic	Ensure all impacts are distributed across social classes equally
	Decommissioning	On-site accident rate	Ensure worker safety
		Regional health impacts	Ensure no adverse health effects on population
		Normalized Theil's T statistic	Ensure all impacts are distributed across social classes equally

Available data limited indicator selection. Data gaps prevented the use of data from a real building, making the example inauthentic. Industrial- or regional-typical values provided estimates for indicators without real data, where applicable. If acceptable values did not exist in the literature, values were assumed for calculation purposes. Table 0.10 compiles all values utilized in the calculations.

Note that the values in Table 0.10 were adjusted from the values given in Table 0.8 to produce a “random” building, with upper and lower values chosen arbitrarily to provide sufficient information to estimate distributions.

Table 0.10. Selected indicator and indicator distribution values for the comprehensive example

Phase	Indicator	High	Mode	Low	Units
Construction	CO_2 emissions	2.00	1.9	1.85	Mt/yr
	Percent recycled materials used	19%	18%	15%	%
	Untreated runoff	5000	4400	4250	lb/(acre-year)
	Percent waste material	14.5%	13%	10%	%
	Road closure impact	7000	5800	5500	Vehicle-hours/day
	Accident rate	0.03	0.02	0	Accidents per day
	Health impacts	2.5%	1.0%	0	% increase
	Normalized Theil's T statistic	0.35	0.15	0.1	Ratio
Operation	CO_2 emissions	78	70	66	Mt/yr
	Energy intensity	42	39.5	38	kBtu/ft^2
	Water usage	9	7	6.5	gal/ft^2
	Untreated runoff	750	675	625	lb/(acre-year)
	Economic impact	0.73%	0.68%	0.62%	% increase
	Property values	0.20%	0.17%	0.16%	% increase
	Accident rate	0.012	0.004	0	Accidents per day
	Health impacts	3.0%	0.9%	0%	% increase
	Normalized Theil's T statistic	0.4	0.1	0.05	Ratio
Decommissioning	CO_2 emissions	1.1	1.0	0.9	Mt/yr
	Materials recycled	88%	85%	80%	%
	Untreated runoff	5200	4800	4200	lb/(acre-year)
	Vehicle impact	7000	6300	5900	Vehicle-hours/day
	Economic impact	-0.1%	-0.25%	-0.3%	% decrease
	Property values	-0.05%	-0.1%	-0.15%	% decrease
	Accident rate	0.04	0.02	0	Accidents per day
	Health impact	0.32%	0.1%	0%	% increase
	Normalized Theil's T statistic	0.4	0.15	0.1	Ratio

Target values are given in Table 0.11, along with sources. Assumptions were used for many indicators, due to a lack of available data. While not realistic, the results demonstrated the viability of the proposed model in a complex application.

Table 0.11. Target values selected for comprehensive example indicators

Phase	Indicator	Target Value	Source
Construction	Bulk CO_2 emissions	20% reduction	(BIS 2010)
	Recycled materials used	15% of purchased materials	(Cyril Sweett 2009)
	Total suspended solids	16 % reduction	Middle Huron Partners and Storm Water Advisory Group (Lawson et al. 2011)
	Percentage waste	10%	Assumed
	Road closure impact	9000 vehicle-hours/day	Assumed
	On-site accident rate	1 per 30 days	Assumed
	Regional health impact	0.5% increase	Assumed
	Theil's T statistic	0.46	Assumed
Operation	Bulk CO_2 emissions	20% reduction	(BIS 2010)
	Energy intensity	45% reduction	(Nelson 2011)
	Water usage	40% reduction	(EPA 2009b)
	Total suspended solids	16% reductions	(Lawson et al. 2011)
	Economic impact	0.6% increase in GRP	Assumed
	Property values	0.15% increase in average property values	Assumed
	On-site accident rate	1 per 45 days	Assumed
	Regional health impact	0.3% increase	Assumed
	Theil's T statistic	0.46	Assumed
Decommissioning	Bulk CO_2 emissions	20% reduction	(BIS 2010)
	Materials recycled	70% - 80%	(Nelson 2011)
	Total suspended solids	16% reduction	(Lawson et al. 2011)
	Road closure impact	9000 vehicle-hours/day	Assumed
	Economic impact	0.35% decrease	Assumed
	Property values	0.1% decrease	Assumed
	On-site accident rate	1 per 30 days	Assumed
	Regional health impact	0.5% increase	Assumed
	Theil's T statistic	0.46	Assumed

Indicator targets came from a variety of sources in order to select semi-realistic numbers. All other inputs were assumed for the purposes of completing the example. The example used "Project valuation and Review Techniques" beta distributions, referred to hereafter as PERT distributions, for all conversion and indicator distributions. A PERT distribution is a method of obtaining a four-parameter beta distribution in a set range. It was chosen to enforce an interval for inputs that could be matched to the selected lower and upper values in the data collection step, but it is not suitable for all indicators in practice. The PERT distribution required an upper bound (x_{max}), a lower bound (x_{min}) and the mode, or "most likely" value (x_{mode}). The equivalent Beta parameters were calculated from the PERT parameters using Eq. 6-14 through 6-19.

$$\alpha_1 = 6\left(\frac{\mu_{PERT} - x_{min}}{x_{max} - x_{min}}\right) \tag{6-14}$$

$$\alpha_2 = 6\left(\frac{x_{max} - \mu_{PERT}}{x_{max} - x_{min}}\right) \tag{6-15}$$

$$f(x|\alpha_1, \alpha_2, x_{min}, x_{max}) = \frac{(x - x_{min})^{\alpha_1 - 1}(x_{max} - x)^{\alpha_2 - 1}}{(x_{max} - x_{min})^{\alpha_1 + \alpha_2 - 1}B(\alpha_1, \alpha_2)} \tag{6-16}$$

$$\frac{1}{B(\alpha_1, \alpha_2)} = \frac{\Gamma(\alpha_1 + \alpha_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \tag{6-18}$$

$$\mu_{PERT} = \frac{x_{min} + 4x_{mode} + x_{min}}{6} \tag{6-19}$$

$B(\alpha_1, \alpha_2)$

$= the\ incomplete\ beta\ function\ with\ beta\ distribution\ parameters\ \alpha_1 and\ \alpha_2$

$\mu_{Pert} = mean\ of\ the\ PERT\ distribution$

The most likely values were taken as typical or target values from the data collection step. A full list of distributions and parameters can be found in Table 0.10 and Table 0.12. All values in Table 0.4 were based on the reductions provided by the defined target values in Table 0.11 applied to the typical values in Table 0.8. Once again, bounds were chosen arbitrarily for the purpose of building the required distributions. For indicators where a lower value corresponded to a higher need probability, the upper tail probability was utilized in the calculation to ensure appropriate indicator interpretation.

Table 0.12. PERT distribution values for conversion distributions for comprehensive example

Indicator	High	Most Likely	Low	Units
CO_2 emissions (Construction)	2.1	2.08	1.8	Mt/yr
CO_2 emissions (Operation)	85	80.56	62	Mt/yr
CO_2 emissions (Decommissioning)	1.12	1.1	0.85	Mt/yr
Percent recycled materials used (Construction)	20%	13%	13%	%
Water usage (Operation)	12	9	6	gal/ft^2
Energy intensity (Operation)	45	43.89	38	kBtu/ft^2
Untreated runoff (Construction and Decommissioning)	5500	5040	4000	lb/acre-year
Untreated runoff (Operation)	900	840	600	lb/acre-year
Materials recycled (Construction)	95%	90%	80%	%
Materials recycled (Decommissioning)	90%	80%	75%	%
Percent waste material (Construction)	15%	10%	2.5%	%
Road closure impact (Construction and Decommissioning)	7500	7000	5000	Vehicle-hrs/day
Economic impact (Operation)	0.75%	0.65%	0.5%	% increase
Property value (Operation)	0.2%	0.15%	0.1%	% increase
Economic impact (Decommissioning)	-0.4%	-0.35%	0%	% decrease
Property value (Decommissioning)	-0.2%	-0.15%	0%	% decrease
Accident rate (Construction and Decommissioning)	0.05	0.03	0	Accidents per day
Health impact (Construction and Decommissioning)	0.04%	0.02%	0%	% increase in claims
Accident rate (Operation)	0.02	0.01	0	Accidents per day
Health impact (Operation)	0.035%	0.02%	0%	% increase in claims
Normalized Theil's T statistic (Construction, Operation, and Decommissioning)	0.4	0.4	0	Ratio

Monetary values were assumed. The base value and expected revenue were $10 million and $15 million, respectively. Consequence values were also measured in millions of dollars. A linear relationship between need probability and consequence was assumed without real data. Table 0.13 contains the maximum and minimum values for the assumed consequence functions.

Table 0.13. Bounds for linear consequence functions for comprehensive example indicators

Phase	Indicator	Minimum	Maximum
Construction	Bulk CO_2 emissions	-0.1	0.3
	Recycled materials used	-0.1	0.2
	Total suspended solids	-0.01	0.1
	Percentage waste	-0.05	0.4
	Road closure impacts	-0.02	0.05
	On-site accident rate	-0.01	0
	Regional health impact	-0.2	0
	Theil's T statistic	0	0
Operation	Bulk CO_2 emissions	-0.3	0.4
	Energy intensity	-0.1	0.3
	Water usage	-0.2	0.4
	Total suspended solids	-0.01	0.1
	Economic impact	0	0.6
	Property values	0	0.4
	On-site accident rate	-0.01	0
	Regional health impact	-0.2	0
	Theil's T statistic	0	0
Decommissioning	Bulk CO_2 emissions	-0.01	0.03
	Materials recycled	-0.01	0.1
	Total suspended solids	-0.1	0.3
	Road closure impacts	-0.02	0.05
	Economic impact	-0.1	0
	Property values	-0.05	0
	On-site accident rate	-0.01	0
	Regional health impact	-0.2	0
	Theil's T statistic	0	0

Correlations accounted for dependence relationships between variables. Accounting for dependencies could be achieved through the full development of marginal and joint probability distributions or statistical estimation procedures. Understanding the true nature of how dependencies behave made accurately eliciting for joint distributions infeasible. Furthermore, attempting to define joint distributions empirically required data that were not readily available. An assumption of linear correlation was selected for the comprehensive example in order to simplify the calculations. Table 0.14 provides the assumed correlations in qualitative terms. In practice, correlations will vary based on the building studied and the region the buildings are located in, among other factors. For this particular case, *Low* corresponded to a correlation of 0.3, *Medium* to 0.5 and *High* to 0.7 for calculation purposes.

Table 0.14. Correlations between comprehensive example indicators

Indicator 1	Indicator 2	Correlation (+ for positive, - for negative)
Bulk CO_2 emissions (construction)	Regional health impact (construction)	+ High
Bulk CO_2 emissions (operation)	Regional health impact (operation)	+ High
Total suspended solids (decommissioning)	Regional health impact (decommissioning)	+ Med
Total suspended solids (construction)	Regional health impact (construction)	+ Med
Bulk CO_2 emissions (decommissioning)	Regional health impact (decommissioning)	+ Med
Total suspended solids (operation)	Regional health impact (operation)	+ Low
Economic Impact (operation)	Property values (operation)	+ Med
Economic Impact (operation)	Property values (decommissioning)	- Low
Economic Impact (operation)	Thiel statistic (operation)	- Low
Economic Impact (decommissioning)	Property values (operation)	- Low
Economic Impact (decommissioning)	Property values (decommissioning)	+ Med
Economic Impact (decommissioning)	Thiel statistic (decommissioning)	+ Low
Property Value (operation)	Thiel statistic (operation)	+ Low
Any relationships not listed here are assumed to have no correlation		

Once all necessary inputs were defined, the simulation procedure previously outlined was followed. The Chi-squared goodness-of-fit served as the criterion for selecting the best fitting distributions from the simulated output. Since the final result was a probability, a 0 to 1 interval was enforced. Equations 6-20 and 6-21 provided the fitted PDF of the final need probability distribution and the consequence distribution, respectively. Consequence valuation underwent a similar procedure; however, no bounds were required for the PDFs.

$$f(p) = \frac{1}{B(1.4734,44.956)} p^{0.4734}(1-p)^{43.956} \tag{6-20}$$

$$f_C(c) = \frac{(c-1.6696)^{11.417}(3.5727-c)^{4.6856}}{(1.9031)^{17.1026}B(12.417,5.6856)} \tag{6-21}$$

Figure 0-1 contains the probability density results for each phase of the building's lifecycle, as well as the overall result.

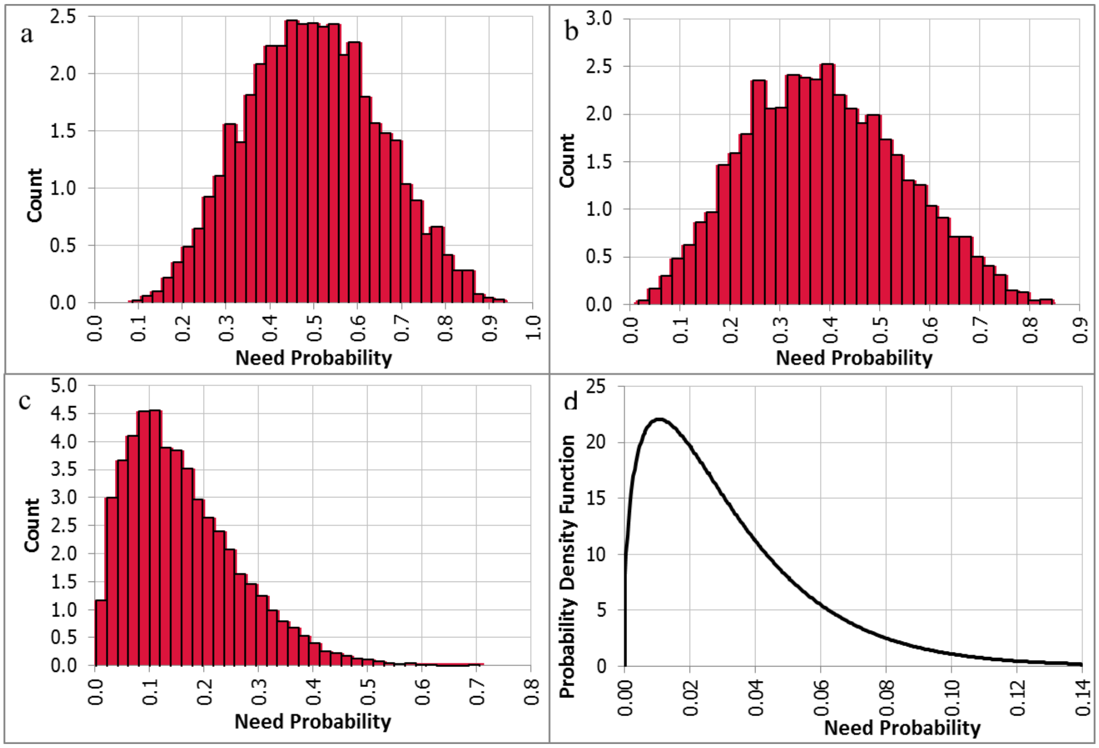


Figure 0-1. Intermediate results: a) construction, b) operation, c) decommissioning and d) final

The marginal distribution for consequence is plotted separately in Figure 0-2.

Table 0.15 contains the statistical moments for the resulting distributions.

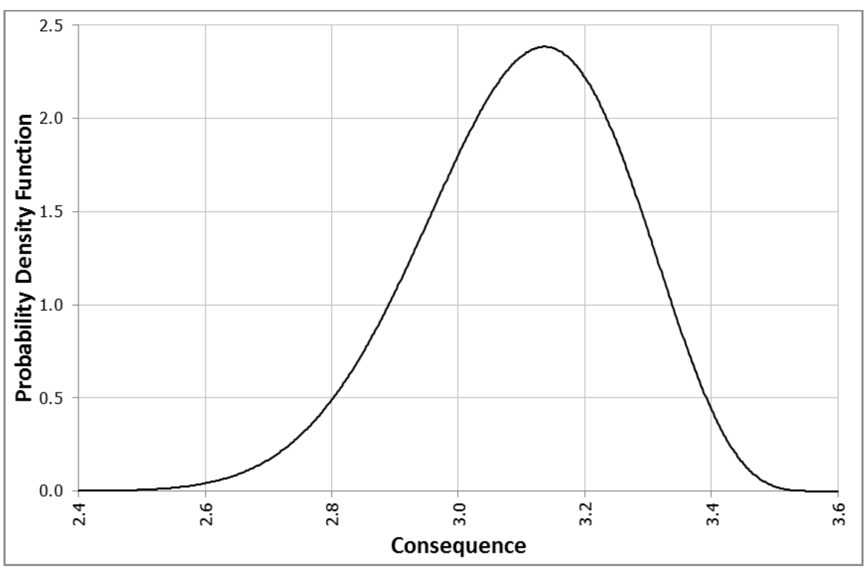


Figure 0-2. Marginal PDF of consequence results from simulation

Table 0.15. Descriptive statistics of total aggregation

Distribution	Mean of need probability	Std. dev. of need probability	Mean of consequence	Std. dev. Of consequence
Total Aggregation	3.24%	2.60%	3.10	0.16
Aggregated Construction Phase	49.99%	15.01%	0.95	0.05
Aggregated Operation Phase	38.97%	15.62%	1.86	0.13
Aggregated Decommissioning Phase	16.24%	10.15%	0.30	0.08

Next, the JPDF was determined. Equation 6-22 provides the functional form of the JPDF assuming independence between need probability and consequence values. The independence assumption here was made to illustrate the methodology. An example involving DSS will follow.

$$f_{C,P}(c,p) = \frac{1}{B(1.4734{,}43.956)} p^{0.4734}(1-p)^{43.956} * \frac{(c-1.6696)^{11.417}(3.5727-c)^{4.6856}}{(1.9031)^{17.1026} B(12.417{,}5.6856)} \tag{6-22}$$

Several analysis techniques can be used once all the pertinent probability distributions have been defined. Equation 6-23 calculates directional cosines, αi, evaluated at the design points. Design points were taken as the “most likely” value from the PERT distribution. Table 0.16 summarizes the results for this example.

$$\alpha_i = \frac{\frac{\partial g(s)}{\delta s_i}\sigma_{s_i}}{\sqrt{\sum_{j=1}^{n}\left(\frac{\delta g(s)}{\delta s_j}\right)^2}}, for\ i = 1,2, \dots, q \tag{6-23}$$

$s = \ placeholder\ for\ an\ indicator$

$q = \ total\ number\ of\ indicators$

Table 0.16. Directional cosines for comprehensive example indicators

Phase	Pillar	Indicator	Directional Cosine
Construction	Environmental	Bulk CO_2 emissions	-0.0179
		Recycled material used	0.00538
		Untreated runoff (TSS)	-0.0585
		Percent waste	0.111
	Economic	Road closure impact	-0.0538
	Social	Accident rate	-0.303
		Health impact	-0.218
		Theil statistic	-0.0171
Operation	Environmental	Bulk CO_2 emissions	-0.0605
		Energy intensity	-0.0113
		Water usage	-0.0685
		Untreated runoff (TSS)	-0.0195
	Economic	Economic impact	0.286
		Property value	0.178
	Social	Accident rate	-0.143
		Health impact	-0.208
		Theil statistic	-0.00450
Decommissioning	Environmental	Material recycled	0.134
		Untreated runoff (TSS)	-0.457
		Bulk CO_2 emissions	-0.188
	Economic	Road closure impact	-0.176
		Economic impact	0.248
		Property value	0.230
	Social	Accident rate	-0.415
		Health impact	-0.273
		Theil statistic	-0.0200

Directional cosines are useful in determining reliability and optimizing performance (Ayyub 2014), but also act as a sensitivity measure of the performance function. For the assumed building, the directional cosine with the largest absolute value corresponded to the untreated runoff indicator in the decommissioning phase. A high absolute value for untreated runoff signified that a slight change in its “most likely” value will have a larger impact than a slight change in other indicators. The negative sign denoted that an inverse relationship existed between the final need probability and the untreated runoff indicator.

Tornado diagrams are another form of sensitivity analysis. A tornado diagram maps the change in an output variable versus incremental changes in input variables. Figure 0-3 presents the tornado diagram for a selected number of input variables with the PERT distribution “most likely” value varied by +/- 2%.

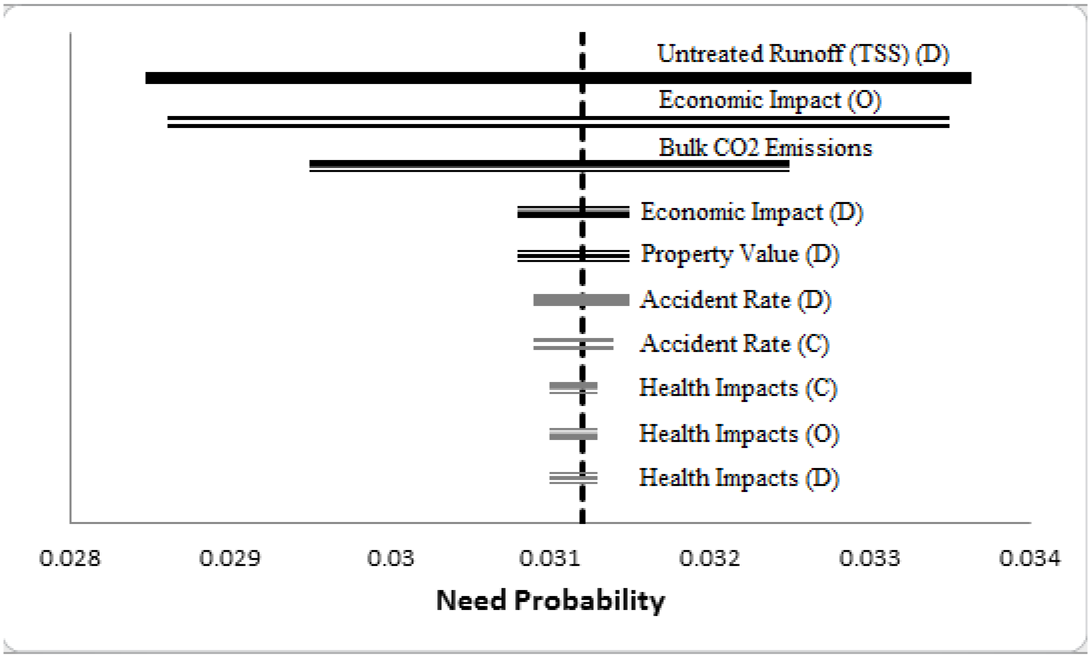


Figure 0-3. Tornado diagram for the 10 results that the need probability was most sensitive to

Ordinarily all input variables are examined; here, the directional cosines were utilized to determine the most sensitive variables. For brevity, Figure 0-3 contains only the ten most sensitive inputs. The importance of tornado diagrams and directional cosines is that both allow for optimization of specific variables.

Benefit-cost ratio calculations were conducted next. Equation 6-24 gives the benefit/cost ratio JPDF with need probability.

$$f_{BCR,P}(bcr,p) = \frac{1}{B(1.4734,43.956)} p^{0.4734}(1-p)^{43.956} * \qquad (6\text{-}24)$$

$$\frac{(bcr-1.7046)^{11.417}(1.858-bcr)^{4.6865}}{(0.1534)^{17.1035}B(12.417,5.6865)}$$

The union of achieving a specific benefit/cost ratio and need probability was found by solving Eq. 6-24 for the benefit/cost ratio and need probability of interest. A benefit/cost comparison of multiple buildings is also viable using the JPDF. Applying the method of comparing need profiles established previously required finding a criterion benefit/cost ratio. The mean was chosen in this case.

$$\mu_{BCR} = \frac{\alpha_1 c + \alpha_2 a}{\alpha_1 + \alpha_2} = \frac{12.417(1.858) + 5.6865(1.7046)}{12.417 + 5.6865} = 1.810$$

Once the mean was found, the need profile in Eq. 6-25 was determined.

$$f_{P,BCR}(p, bcr \geq 1.81) = 0.527 \frac{1}{B(1.4734,43.956)} p^{0.4734}(1-p)^{43.956} \tag{6-25}$$

The mean need probability when the benefit/cost ratio was greater than or equal to its mean was calculated as $\mu_{P,BCR}$, where

$$\mu_{P,BCR} = 0.527\left(\frac{1.4734}{43.956 + 1.4734}\right) = 0.0171$$

A comparison between two buildings could then proceed as in basic example 1, using $\mu_{P,BCR}$ or the mean benefit/cost ratio alone.

Figure 0-4 contains a plot of the spider diagram for the analyzed building in this comprehensive example, incorporating the probabilistic nature of the results.

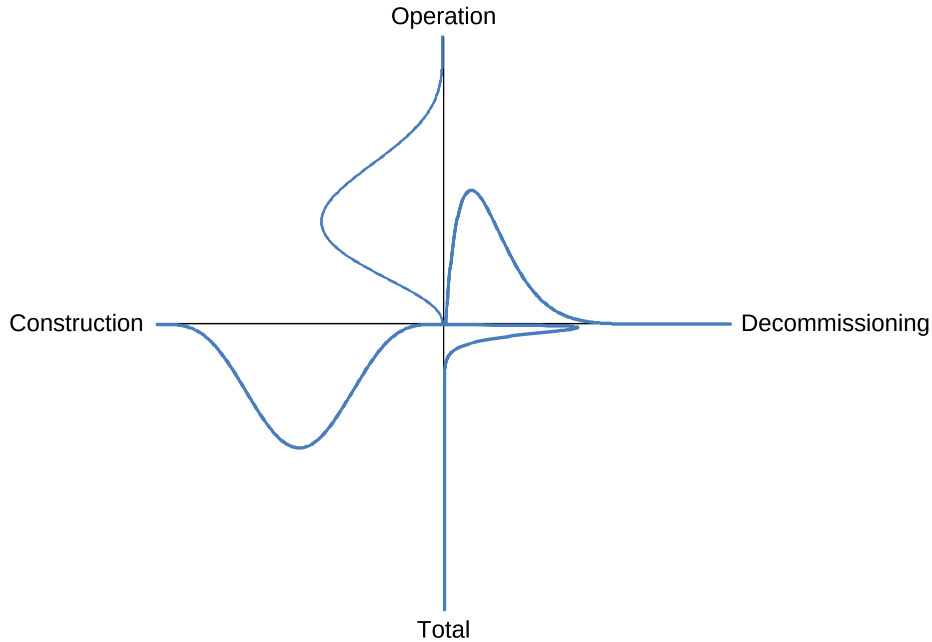


Figure 0-4. Spider diagram of example results including uncertainty

Each axis plots a PDF, and therefore represents the distribution of need probability for that phase. The main purpose of the spider diagrams is to facilitate the comparison of an alternative with a set of targets. For illustration purposes, the PDFs of need probability for the construction, operation and decommissioning phases of the building, denoted *Building A*, are presented in Eq. 6-26 to 6-28, respectively. The subscript A indicates that the distribution refers to Building A. Subscripts C, O and D refer to the need probabilities of the construction, operation and decommissioning phases, respectively.

$$f_{P_{C_A}}(p_{C_A}) = \frac{1}{B(5.0485,5.0513)} {p_{C_A}}^{4.0485} (1 - p_{C_A})^{4.05131} \quad (6\text{-}26)$$

$$f_{P_{O_A}}(p_{O_A}) = \frac{1}{B(3.4102,5.3401)} {p_{O_A}}^{2.4102} (1 - p_{O_A})^{4.3401} \quad (6\text{-}27)$$

$$f_{P_{D_A}}(p_{D_A}) = \frac{1}{B(1.9083,10.215)} {p_{D_A}}^{0.9083} (1 - p_{D_A})^{9.215} \quad (6\text{-}28)$$

The total need probability PDF was previously defined in Eq. 6-20. Now, a second building, denoted Building B, with the construction (Eq. 6-29), operation (Eq. 6-30) and decommissioning (Eq. 6-31) need probability PDFs will be compared to Building A. Subscript B denotes that the distribution refers to the second building, *Building B*, under consideration.

$$f_{P_{C_B}}(p_{C_B}) = \frac{1}{B(4,3.5)} p_{C_B}^{3} (1 - p_{C_B})^{2.5} \quad (6\text{-}29)$$

$$f_{P_{O_B}}\left(p_{O_B}\right) = \frac{1}{B(3,6)} p_{O_B}^{2.4102}\left(1 - p_{O_B}\right)^{4.3401} \qquad (6\text{-}30)$$

$$f_P\left(p_{D_B}\right) = \frac{1}{B(2,8)} p_{D_B}^{0.9083}\left(1 - p_{D_B}\right)^{9.215} \qquad (6\text{-}31)$$

The total need probability distribution for Building B is given in Eq. 6-32.

$$f_{P_B}(p_B) = \frac{1}{B(1.3550,37.727)} p_B^{0.3550}(1 - p_B)^{36.727} \qquad (6\text{-}32)$$

Figure 0-5 plots Building A and Building B on a single spider diagram.

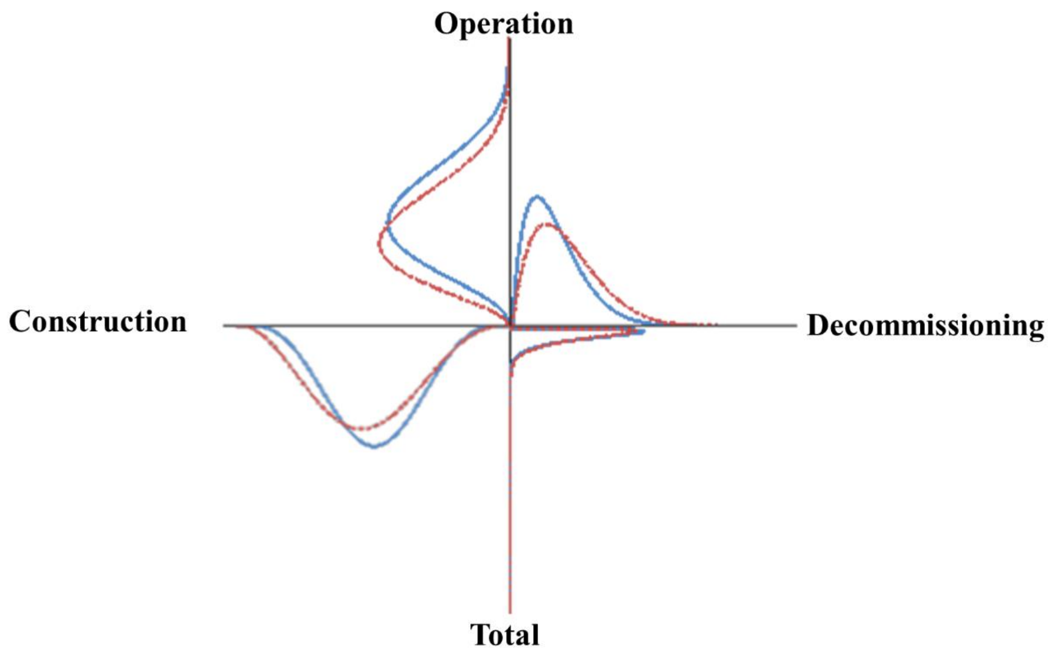


Figure 0-5. Spider diagram comparing the results of the comprehensive example to a hypothetical second alternative. Blue - Building A, Red - Building B

By overlaying the distributions of the alternative, it is possible to visually inspect for any obvious dominance of one over the other. Due to the probabilistic nature of the methodology's results, it became difficult to select an alternative by inspection alone. The easiest way to compare alternatives in such a case is by examining stochastic dominance (Clemen and Reilly 2014). Using the previously described simulation method with two thousand cycles, the probabilities of Building A outperforming Building B by phase, and in total, are given below.

$$Prob_{Construction}(P_1 > P_2) = 0.098$$

$$Prob_{Operation}(P_1 > P_2) = 0.611$$

$$Prob_{Decom}(P_1 > P_2) < 0.000$$

$$Prob_{Total}(P_1 > P_2) = 0.215$$

Based on the calculations, Building B outperformed Building A in all phases except operation. In total, the need probability of Building B had a 0.785 probability of being more sustainable than Building A.

Alternatively, the PDF for a portfolio of multiple buildings may be of interest. The JPDF for the portfolio was calculated as in basic example 1 for two identical and independent buildings in Eq. 6-33.

$$f_{C_1,C_2,P_1,P_2}(c_1,c_2,p_1,p_2) = f_P(p) = \frac{1}{B(1.4734,43.956)} p^{0.4734}(1-p)^{43.956} * \left(\frac{1}{B(1.4734,43.956)} p^{0.4734}(1-p)^{43.956} \frac{(c-1.6696)^{11.417}(3.5727-c)^{4.6856}}{(1.9031)^{17.1026}B(12.417,5.6856)}\right) * \frac{(c-1.6696)^{11.417}(3.5727-c)^{4.6856}}{(1.9031)^{17.1026}B(12.417,5.6856)} \tag{6-33}$$

For brevity, no further analysis on the joint distribution was reported, however all previous calculations in the example are adaptable to the more complicated distribution for multiple buildings.

6.5.2 The Comprehensive Example Using Dempster-Shafer Structures

The results for the comprehensive example up to the generation of the marginal distributions were identical, regardless of the dependence assumptions. Directional cosines and utilities from the independent case were also unchanged. As such, the variation of the comprehensive example began after the marginal distributions were determined through simulation and will not include any repetitive results.

For this example, an assumption of unknown positive dependence was selected. The reasoning was that additional sustainability measures should produce a more efficient building, a healthier populace and attract a larger market share. Since the base value incorporated the actual cost of implementing the sustainability measures, the consequence values only focused on the impact of the measure once implemented. A joint distribution of consequence and need probability under the assumption of unknown positive dependence required the construction of a DSS. The consequence and need probabilities were divided into 10 equal intervals. The size of the resulting DSS table made it difficult to display efficiently. Figure 0-6 presents a heat map of need probability in lieu of the DSS.

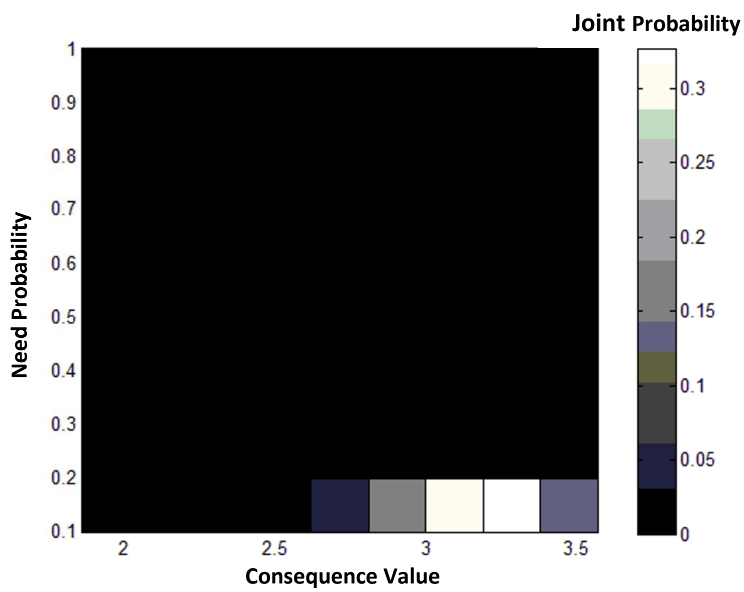


Figure 0-6. Heat map of the DSS structure for the comprehensive example. Upper bound was used for the scale on the color bar.

A heat map is a representation of a three-dimensional plot using a color-coded two-dimensional surface. To obtain a probability from the heat map, the color of a cell in the plot was compared to the “color bar” on the right of Figure 0-6. The color bar was coded to the upper bound of the probability interval for each interval box in Figure 0-6, however the mean of the probability interval or lower bound would be equally valid.

Obtaining exact probabilities from the heat map was difficult, due to the use of color coding. The only meaningful value that was obtainable was an estimate of the upper bound being in a particular interval box. The CDF can be bounded using the DSS table that stores the data for the plot. An example of such a calculation is presented later.

Obtaining the benefit/cost ratio values for analysis required redefining the DSS in terms of benefit cost ratio, as in Figure 0-7.

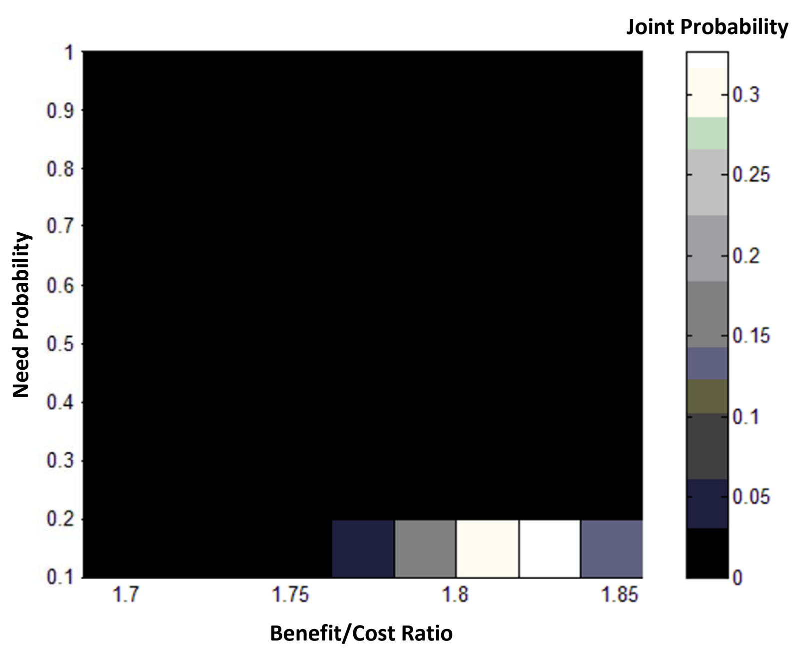


Figure 0-7. Heat map of the DSS structure for the comprehensive example in terms of BCR. Upper bound was used for the scale on the color bar.

The median benefit/cost ratio range, [1.7149, 1.8108], is calculated from the stored DSS table. Comparing the bounds of the median value to those of another building is not intuitive, though the mean of the range is a possible alternative to probability bounds.

If the need profile was considered, the procedure followed the exact same steps as in basic example 2. Analyzing the DSS to identify where the majority of the need probability was located yielded the following upper bounds: [0.1, 0.12, 0.14, 0.16, 0.18, 0.2]. Table 0.17 provided the resulting need profile for the median range.

Table 0.17. Need profile Dempster-Shafer structure for the median benefit/cost ratio interval

P	BCR [1.715,1.811] 0.499
[0,0.01] 0.179	[0.179,0.360]
[0,0.02] 0.392	[0.392, 0.787]
[0,0.03] 0.568	[0.568,1.0]
[0,0.04] 0.701	[0.701,1.0]
[0,0.06] 0.864	[0.864,1.0]
[0,0.1] 0.976	[0.976,1.0]

Determining a median value for the need probability, given the median consequence interval, aided in comparing the alternative to other buildings. The lower bound of the median range was taken as the upper bound of the last interval that was too small to contain the median; i.e. 0.01. The upper bound of the median was the upper bound of the first consequence value that was too large to be the median; i.e. 0.03 in this example. The average of this range, 0.02, may be more useful for a practical comparison of one building to another.

As an illustration of how to create a joint distribution for multiple buildings, a second building was considered with the marginal distributions for need probability and consequence given in Eq. 6-34 and 6-35, respectively.

$$f_P(p) = \frac{1}{B(1.6,34)} p^{0.6}(1-p)^{33} \tag{6-34}$$

$$f_C(c) = \frac{(c-1.5)^{18}(3.8-c)^{7}}{(2.3)^{26}B(19,8)} \tag{6-35}$$

Assuming unknown positive dependence between buildings and utilizing the prescribed methodology, a joint distribution of both buildings was estimated by first dividing each marginal distribution into 10,000 intervals. Due to its size, the resulting DSS is omitted from this paper. For calculation purposes, it was assumed that the probability of the consequence value being less than six was of interest. Using a table look-up algorithm on the DSS, the following probability bounds were obtained.

$$Lower\ Bound = [0.1603{,}0.2902]$$

$$Upper\ Bound = [0.6526{,}0.9845]$$

The bounds were simplified to an interval between the lowest lower bound and the largest upper bound as well as an interval of the average of the bounds.

Widest bounds:

$$P(C \leq 6) = [0.1603, 0.9845]$$

Average bounds:

$$P(C \leq 6) = [0.225, 0.819]$$

Repeated application of the unknown positive dependence assumption increased the uncertainty in the calculation until the bounds became almost meaningless. In practice, if the strength of the dependence can be estimated, the methodology becomes far more useful, allowing for a value to be selected from the range with more confidence. Otherwise, the results offer little insight.

6.5.3 Comprehensive Example Discussion

The comprehensive example illustrated the proposed model in a semi-realistic context. The large amount of initial data required for this example revealed the importance of a rigorous data collection scheme for implementation. The simulation process remained straightforward when using more complex data, and the fit distributions were useful in identifying problematic phases allowing for targeted solutions. The use of probability made this methodology adaptable and versatile in terms of analysis. This flexibility allows practitioners to apply it beyond the selected analysis of the comprehensive example. Methods to calculate tradeoffs, incorporation of full lifecycle cost assessments and engineering economic applications are all available under the proposed methodology. Figure 0-7 provides an illustration of the method's flexibility. Incorporating the probabilistic results into the familiar spider diagram also shows the ease with which the results can be implemented using preexisting methods.

A variant of the comprehensive example would be useful in understanding where the primary differences lie between the independent and dependent cases. The use of bounds reduced the probabilities within the CDF to ranges. Utilizing ranges is not as straightforward as utilizing the results under the independence assumption. Furthermore, the repeated application of interval boxes increased probability bounds to meaningless ranges. This illustrates why uncertainty should not simply be omitted in such calculations. Increasing the number of interval boxes can reduce this effect, but it comes at the cost of increased computation time. Once a satisfactory range is achieved, a number of analysis methods can be used to obtain a single value by fitting a distribution to the bounded probability range based on engineering judgment. In some cases, the number of intervals required to achieve a satisfactory result from the use of Dempster-Shafer structures may prove to be too cumbersome.

Chapter 7: Non-parametric Model

A non-parametric model was developed in an attempt to resolve some of the issues with the use of DSSs. The primary aim was to remove the need to report the results as ranges of values while maintaining as much information on the uncertainty in the output as possible.

7.1 The Reasoning behind the Non-Parametric Model

The goal of the non-parametric model is two-fold: first, to show that the general methodology is adaptable to the manufacturing domain, and second, to present a method for constructing the joint distributions that is both easier to understand and capable of producing more meaningful results in practice. To achieve this goal, a hypothetical manufacturing process was developed and examined using the proposed methodology. The following section is divided into subsections corresponding to the individual steps presented in the Figure 0-1 outline.

The primary change from the methodology established previously is in the way distributions are constructed. While interesting from an academic standpoint, DSS is not as useful in practical applications. One way to address the issues of DSS is to leverage the fact that the proposed methodology relies on simulation.

Simulation techniques allow the analyst to obtain as many results as desired, with convergence to some value as the termination criterion. While calculation time and memory are both limiting factors, a simulation theoretically allows for an interminable number of cycles to be run. The methodology herein leverages the theoretically unbounded nature of simulation cycles to solve some of the issues arising from the use of DSS.

Consider the following set of data pairs in Table 0.1, sorted by the x-column, that was assumed to result from a simulation.

Table 0.1. Example data set, sorted by X

x	y
1.2	2.6
1.8	5.3
2.1	3.1
2.3	4.6
2.6	2.9
3.1	6.2
3.2	5.4
3.5	3.2
4.1	2.7
4.9	5.2

The first benefit is that determining an estimate of the joint probability becomes a counting exercise. For instance, if the joint probability that “x” is less than 3 and “y” is less than 3 is desired, all that is required is to count how many data points meet the criteria (in this case two data points) and then divide that by the total number of data points (10). Therefore the probability that “x” is less than three and “y” is less than three is 0.2. Provided a sufficient number of simulations have been run, the estimate can be determined to a high degree of accuracy, subject to the accuracy of the input data.

The second benefit lies in defining the conditional distribution. If the x-column is taken by itself, then a non-parametric distribution can be built using rank-adjustment or Kaplan-Meier to determine the probabilities. A non-parametric distribution for the y-column is also possible using the same sorting method. But what if a conditional distribution is of interest? In that case, the x-column could be divided into equal blocks of 20% of the data; for instance, the values less than the 20th percentile are given in Table 0.2. Using this method means the uncertainty of estimates will be lower for larger percentiles; for example, the 80th percentile for the points in Table 0.1 would have eight points to base estimates off of instead of only two. Differing levels of uncertainty, based on where in the distribution a point lies, are not unusual in simulation, as the tails of any simulated result will typically not be as well defined. By increasing the number of simulations, the tails of the distribution could theoretically reach an arbitrary confidence bound width, though the extra simulation time may not be justifiable, nor will the tails ever be defined as well as the center of the distribution.

Table 0.2. 20th percentile of data from Table 0.1

x	y
1.2	2.6
1.8	5.3

Any value less than or equal to 1.8 would correspond to a 20% value of the cumulative distribution function. If the x-column in Table 0.2 is treated as only defining the 20th percentile, then the y-column represents all possible values that could arise given that the x-value is less than or equal to 1.8. Thus, a conditional probability distribution could be estimated from the data set. For example,

$$P(Y \leq 4.5 | X \leq 1.4) \cong 0.5$$

where $P(y \leq 4.5 | x \leq 1.4)$ is the probability that the "y" variable is less than or equal to 4.5 given that the "x" variable is less than or equal to 1.4.

The issue that typically limits the use of non-parametric distributions is that the level of refinement in defining a probability distribution is heavily dependent on the size of the dataset available. However, in the case of simulation, the "dataset" being used can be arbitrarily large, subject to calculation time and memory constraints. Therefore, it is possible to obtain an arbitrary level of refinement in a non-parametric estimate of an output distribution. This user-determined level of refinement, along with a non-parametric distribution's insensitivity to input or output distribution types or dependencies, makes it possible to estimate a single probability for the conditional distribution with a high degree of confidence. Note that increasing the number of simulations does not necessarily increase the accuracy of any estimate, only the confidence that the results are an accurate representation of the distribution, given the input variables. Thus, it is possible to have high confidence that the output result is the correct distribution given the inputs, but have the final result be wholly inaccurate due to a lack of, improper inclusion of or the erroneous nature of some or all of the inputs.

The benefit of the ability to define the conditional distribution lies in the definition of the joint distribution in Eq. 7-1:

$$f_{X,Y}(x, y) = f_X(x) f_{Y|X}(y|x) \tag{7-1}$$

where $f_{X,Y}(x, y)$ is the joint probability density function (PDF) of "x" and "y", $f_X(x)$ is the marginal PDF of "x" and $f_{Y|X}(y|x)$ is the conditional PDF of "y" given "x". The marginal can also be written as in Eq. 7-2.

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x, y)}{f_X(x)} \tag{7-2}$$

In the context of the methodology presented here, the average value of "x", denoted $\bar{x}$, was calculated from the marginal distribution of "x". From Eq. 7-3, the conditional mean of "y" is then:

$$E(y|x \leq \bar{x}) = \int_{-\infty}^{y} \int_{-\infty}^{\bar{x}} y \frac{f_{X,Y}(x, y)}{f_X(x)} dx\, dy \tag{7-3}$$

where $E(y|x \leq \bar{x})$ is the expected value of "y" given that "x" is less than or equal to the mean value of "x", denoted by "$\bar{x}$".

Typically, Eq. 7-3 requires a functional form of the joint distribution $f_{X,Y}(x, y)$, but by using the non-parametric data, the process becomes a summation of individual data points in lieu of an integration.

Another benefit is the ability to generalize the calculation into a "what if" scenario based on setting a single input. For instance, "what would the average value of “y” be if the “x” value is at the 40th percentile?” This is determined by sorting “x” by the possible values of “y,” now easily obtained using Eq. 7-4.

$$\{y|x \leq x_{40}\} = \{2.6, 5.3, 3.1, 4.6\} \tag{7-4}$$

where $\{y|x \leq x_{40}\}$ is the set of all values of “y” given that “x” is less than or equal to the 40th percentile of “x,” denoted “x_{40}”.

Using non-parametric methods produces a quick estimate of the distribution of “y.” The drawbacks of utilizing non-parametric methods are well known. The first is that prediction outside the bounds of the generated dataset is impossible, though the possibility of this issue arising can be significantly reduced by increasing the number of simulations. Another drawback lies in uncertainty characterization. When using maximum-likelihood, uncertainty bounds fall naturally outside of the calculation of the Fisher information matrix. Uncertainty bounds for non-parametric estimates must then be constructed. In some cases, percentile rank can be used to estimate the upper and lower bounds of a result; however, this is not always possible. In situations where ranked percentiles fail, bounds are constructed using the estimated mean and variance in the typical confidence bound calculation, as in Eq. 7-5.

$$x \pm t\left(\frac{\alpha}{2}, \mathrm{dof}\right) * \mathrm{Var}(x) \tag{7-5}$$

where $t\left(\frac{\alpha}{2}, \mathrm{dof}\right)$ is the value of the student's t-distribution for a lower tail probability of $\frac{\alpha}{2}$ with degrees of freedom, dof, and $\mathrm{Var}(x)$ is the variance of the "x" variable.

While commonly used, Eq. 7-5 implicitly assumes that the error in "x" is normally distributed. If the normality assumption fails, bootstrapping methods are typically implemented.

Further complicating uncertainty characterization is the multilayer nature of the conditional distribution. The uncertainty in any estimated value from the conditional distribution includes both the uncertainty in the value determined at the conditional level and the uncertainty in the value at the marginal level. To effectively account for this compounded uncertainty, the marginal level uncertainty can be estimated using non-parametric methods, such as bootstrapping. The marginal level bound's conditional level uncertainty can then be calculated in addition to the point estimate conditional level uncertainty.

The most prevalent issue in this case is the large number of simulations required. Assuming that a degree of refinement is desired such that there are at least 100 data points for each 1% percentile at the conditional level, the required number of simulations would be 1,000,000. Depending on how involved the simulations are, the simulation process could become excessively time consuming and memory issues could arise. Furthermore, based on the nature of the inputs, there is no guarantee that the resolution is sufficient to ensure an accurate estimate at the conditional level. It should also be noted that accurate estimates may be possible with significantly less data points.

7.2 Illustration of the Non-Parametric Model

To illustrate the non-parametric methodology, an example manufacturing process from the steel industry was constructed. The example herein was not based on a real manufacturing operation and was instead purposefully created to be easy to follow and validate. As much as possible, values were chosen based on typical data. In some cases, values were assumed for the purposes of filling in data gaps necessary for creating a calculable example.

The example that follows is meant to be illustrative of the process, and the results should not be considered meaningful in numeric context. This is the result of assumptions made out of necessity, or in order to simplify the analysis to aid in the illustrative nature of the examples. In general, typical values were obtained from literature where applicable for the indicator distributions. Target distributions were generated from published works where possible and assumed when no values could be found. Uncertainty was added under assumed distribution types and with assumed coefficients of variation and correlations. Consequence functions were assumed linear. Because of the illustrative nature of the examples, no sensitivity analysis was performed on the inputs.

7.2.1 Select Applicable Lifecycle Phases and Sustainability Pillars

For the purposes of this example, all three pillars were considered. In determining lifecycle phases to consider, a concession was made so that the example was easy to follow. The full steel manufacturing process from cradle to grave contains many suppliers, distributers and other intermediate steps that make for a highly complex system. While the proposed methodology was designed to be flexible enough to accommodate such complexity, it was inappropriate to include for the purposes of an illustrative example. As such, only three phases were considered: 1. extraction of iron ore (i.e. mining operations), 2. steel production (including intermediate product production such as coke) and 3. steel fabrication.

7.2.2 Select Indicators

For the non-parametric example, the indicator groupings in Table 0.3 were selected. This list should not be considered complete, nor are the results representative of an actual process. The selected indicators and values that follow were chosen solely to provide an example application of the methodology.

Table 0.3. Selected phases and indicators for the manufacturing example

Manufacturing phase	Sustainability pillar	Indicator
Ore extraction	Environmental	Greenhouse gas emissions
		Effluent (total metals)
	Economic	Investment in new products and procedures
	Social	Work related fatalities, injuries and illnesses
Steel production	Environmental	Greenhouse gas emissions
		Energy intensity
		Material intensity
	Economic	Investment in new products and procedures
		Economic value distributed
	Social	Lost time injury frequency rate
		Employee training
Steel fabrication	Environmental	Greenhouse gas emissions
		Non-renewable primary energy demand
		Acidification potential
	Economic	Investment in new products and procedures
		Economic value distributed
	Social	Work related fatalities, injuries and illnesses

It should be noted that the difference in indicators between phases was driven primarily by the availability of the data. In some instances, environmental impact factors from LCAs were used as indicators.

7.2.3 Define the Sustainability Need that Each Indicator Should Inform

Table 0.4 contains the definitions of the sustainability goals for the steel example.

Table 0.4. Sustainability goals for each indicator

Indicator	Sustainability goal
Greenhouse gas emissions	Limiting global warming impact
Energy intensity and Non-renewable primary energy demand	Ensuring efficient use of energy
Material efficiency	Ensuring efficient use of materials through recycling and reuse
Acidification potential	Limiting the acidification of rain and waterways
Effluent (total metals)	Limiting the adverse effects of heavy metals on local waterways. Note that heavy metals are typically broken down to individual level outputs, but are aggregated here for simplification
Investment in new products and procedures	Increasing the efficiency of overall processes and creating more sustainable products
Economic value distributed	Ensuring financial viability through distribution of economic value to stakeholders and re-investment
Work related fatalities, injuries and illnesses	Ensuring workers are safe and healthy
Employee training	Ensuring workers are educated in how to do jobs safely and effectively

7.2.4 Collect Data to Describe Indicators for Simulation Purposes

Table 0.5 presents the indicator distribution data. For data sources that did not provide a minimum and maximum estimate, a range was assumed based on an arbitrary percentage reduction and increase, respectively (bold items in the table). Some values were also changed from those reported in the original sources to produce results that were easier to illustrate. The data were chosen to construct a realistic example illustration of the methodology. Triangular distributions were constructed for all variables from the data in

Table 0.5. The choice of the triangular distribution for all variables was arbitrary.

Table 0.5. Values used for indicator distributions in the manufacturing example

Phase	Pillar	Indicator	Min	Most likely	Max	Units
Ore extraction	Environmental	**Greenhouse gasses***	0.70%	1%	1.10%	% of national emissions†
		Tailings and waste rock*	113490	126100	151320	tonnes†
	Economic	**Research and development (funding)***	$95.0	$100	$115.0	millions†
		Research and development (personnel)*	225	250	275	full-time equivalents†
	Social	**Injuries (non-fatal)***	600	750	900	per 100,000 employees
Steel production	Environment	**Greenhouse gasses****	1.805	1.9	1.995	tonnes CO_2/tonne crude steel cast
		Energy intensity**	18.36	20.4	21.42	GJ/tonne crude steel cast
		Material efficiency**	0.96624	0.976	0.98088	% material converted to products and byproducts
	Economic	**Investment in new processes and products****	7.5075	7.7	7.8925	percentage of revenue
		Economic value distributed**	0.72975	0.973	0.98273	percentage of revenue
	Social	**Lost time injury frequency rate****	1.12	1.4	1.68	injury/million hours worked
		Employee training**	5.2	6.5	7.8	training days per employee
Steel fabrication	Environment	Global warming potential***	0.193	0.215	0.261	kg CO_2-equivalent/kg steel
		Non-renewable energy demand***	1.5	2	2.7	MJ/kg steel
		Acidification potential***	0.0461	0.0519	0.0595	mol H+ equivalent/kg steel
	Economic	**Research and development (funding)***	$2.2	$2.4	$2.7	millions†
		Research and development (personnel)*	42	47	52	full-time equivalents†
	Social	**Injuries (non-fatal)*******	1.376	1.72	2.064	total recordable incident rate

*From Energy and Mines Ministers (2013)
** From World Steel Association (2015)
*** From Weisenberger (2010)
**** From ISN Software Corporation (2013)
† Value in source represented total for all reporting entities. Total value from report was used directly, assuming

that the hypothetical process was representative of the whole industry.

The data for the "need probability" distributions are presented in Table 0.6. All values were assumed for convenience and thus have no basis in literature, regulation or industry. Either ad hoc weighting or adjustment of the conversion distributions themselves could be used to account for the relative importance of each indicator. For the presented analysis, the values in Table 0.6 were assumed to have already been corrected for relative importance.

Table 0.6. Values for need probability distributions in the manufacturing example

Phase	Pillar	Indicator	Distribution	First parameter	Second parameter	Third parameter	Fourth parameter
Ore extraction	Environ.	Greenhouse gasses	Log-normal*	-4.6318	0.05125	-	-
		Tailings and waste rock	Normal**	120000	50000	-	-
	Econ.	Research and development (funding)	Normal**	95	4	-	-
		Research and development (personnel)	Normal**	240	10	-	-
	Social	Injuries (non-fatal)	Normal**	725	25	-	-
Steel production	Environ.	Greenhouse gasses	Log-normal*	0.6113	0.08865	-	-
		Energy intensity	Normal**	20	1	-	-
		Material efficiency	4-parameter Beta***	2.2727	3.7273	0.89	1
	Econ.	Investment in new processes and products	Log-normal*	1.9739	0.01830	-	-
		Economic value distributed	4-parameter Beta***	4.3333	1.6667	0.7	1
	Social	Lost time injury frequency rate	Log-normal*	0.2545	0.1256	-	-
		Employee training	Log-normal*	1.9091	0.02927	-	-
Steel fabrication	Environ.	Global warming potential	Normal**	0.195	0.1	-	-
		Non-renewable energy demand	Normal**	2.8	0.5	-	-
		Acidification potential	Log-normal*	-2.9979	0.06574	-	-
	Econ.	Research and development (funding)	Log-normal*	0.8751	0.02745	-	-
		Research and development (personnel)	Log-normal*	3.8261	0.07142	-	-
	Social	Injuries (non-fatal)	Log-normal*	0.4016	0.08748	-	-

* First parameter – Log-mean; Second parameter – Log-standard deviation
** First parameter – mean; Second parameter – Standard deviation
*** First parameter – Shape factor α_1; Second parameter – Shape factor α_2; Third parameter – Lower bound; Fourth parameter – Upper bound

Table 0.7 lists the assumed consequence functions for the proposed example. For simplicity, all consequence functions were assumed to be linear as a function of the indicator level. The consequence functions were assumptions and were not taken from the literature, industry or regulations. The time-value of money was not considered in order to simplify the example; however, the simulation of cash flows, or an assumption of uniform cash flow based on the total simulated value, could easily be used.

Table 0.7. Values for consequence functions in the manufacturing example

Phase	Pillar	Indicator	Slope	Intercept	Units
Ore Extraction	Environmental	Greenhouse gasses	40	-0.4	$/tonne
		Tailings and waste rock	1.64E-05	-2.067213	$/tonne
	Economic	Research and development (funding)	0.03	-3	$/tonne
		Research and development (personnel)	0.01	-2.5	$/tonne
	Social	Injuries (non-fatal)	0.004	-3	$/tonne
Steel Production	Environmental	Greenhouse gasses	1	-1.9	$/tonne
		Energy intensity	0.025	-0.51	$/tonne
		Material efficiency	-1.69173	1.651128	$/tonne
	Economic	Investment in new processes and products	-0.25	1.925	$/tonne
		Economic value distributed	-4.43787	4.318047	$/tonne
	Social	Lost time injury frequency rate	0.5	-0.7	$/tonne
		Employee training	0.4	-2.6	$/tonne
Steel Fabrication	Environmental	Global warming potential	3.75	-0.80625	$/tonne
		Non-renewable energy demand	0	0	$/tonne
		Acidification potential	26.31579	-1.365789	$/tonne
	Economic	Research and development (funding)	0.425	-1	$/tonne
		Research and	-0.05	2.35	$/tonne

		development (personnel)			
	Social	Injuries (non-fatal)	0.227273	-0.390909	$/tonne

The base cost represented the net present value of implementing an alternative. To separate the consequence of a particular indicator level from the cost of implementing a particular sustainability measure in order to achieve it, the cost of sustainability measures was included in the base cost of the product. Revenue was treated separately from the base cost and represented the net present value of the expected income generation for the product, excluding the consequences. Both the base cost and revenue were treated as probabilistic for the purposes of simulation, and were considered determinant values for this example. Table 0.8 outlines the base cost and revenue for the example, as well as the sources for the data. To show how combined costs could be handled by the methodology, it was assumed that the same company owned all three phases in the example, though it is acknowledged that this is unrealistic.

Table 0.8. Base cost and revenue values for the manufacturing example

Phase	Amount	Source
Mining expenses	$25 per tonne	(Gilroy 2014)
Freight – mine to mill	$11 per tonne	(Gilroy 2014)
Mill expenses	$866 per ton	(Commercial Metals Company 2002)
Selling price	$907 per tonne*	(Commercial Metals Company 2002)
* Additional $36 (mining expenses plus freight costs) added to the amount calculated from the source to account for the fact that the original source was for a mill only, making the additional mining and freight costs produce a net loss per ton steel.		

7.2.5 Synthesize Probability Distributions and Consequence Functions

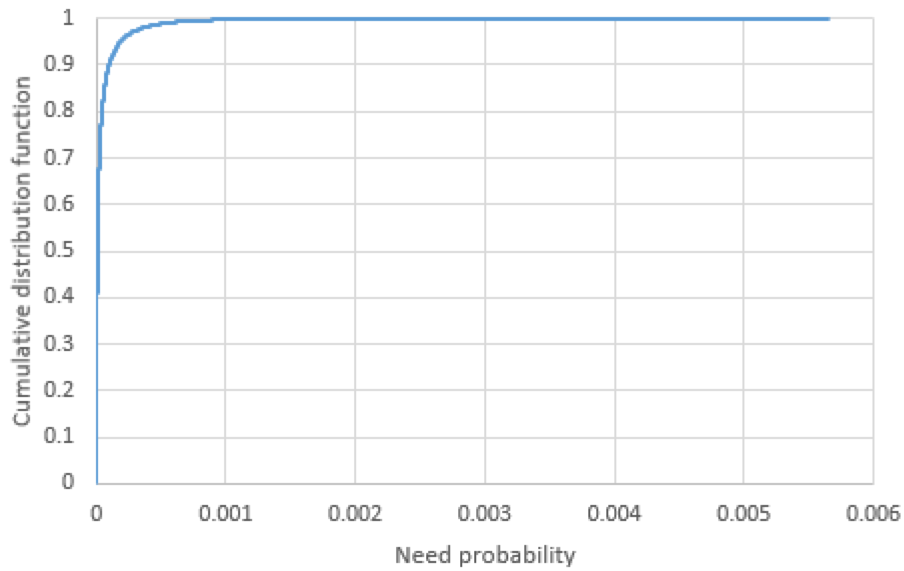
The results of this step have been included in section 7.2.4. This section was only retained to ensure subsequent section numbering adhered to that defined in Section 6.1.

7.2.6 Run Simulations

The simulation procedure was previously presented in Figure 10. 100,000 simulation cycles were used, corresponding to at least 10 data points for each percentile at the conditional level.

7.2.7 Fit Probability Density Functions to the Need Probability and Consequence Results of Interest

In the original methodology this process involved the use of distribution fitting to obtain parametric results. The current example uses non-parametric results, requiring an alteration to this step. Figure 0-1 contains the non-parametric CDFs for the marginal need probability and consequence distributions. The sharp increase seen in the need probability CDF was due to most of the results lying near zero. This would indicate poor performance; however, the arbitrary nature of the conversion distributions makes it impossible to tie real world significance to the numerical results. What matters is that the CDF appeared smooth, indicating that, for the marginal at least, the simulation provided a fairly good representation of the entire distribution.



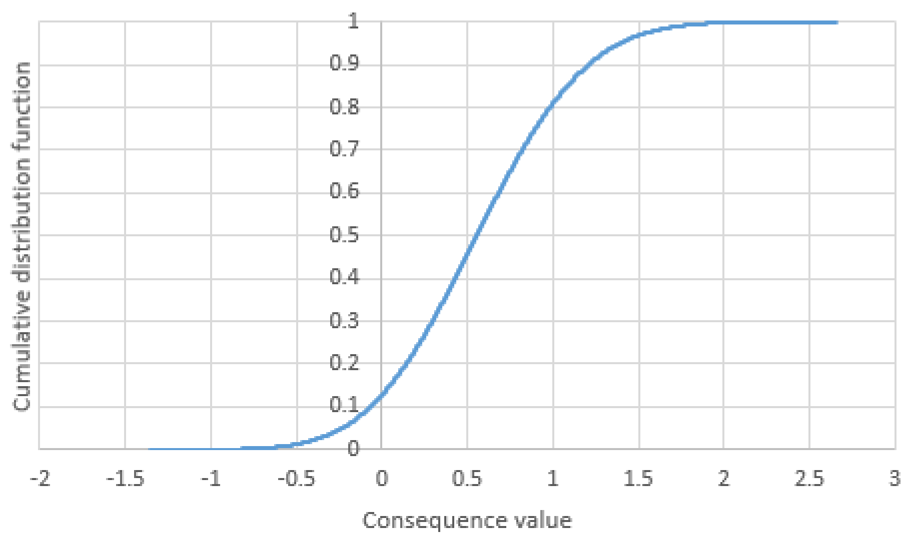


Figure 0-1. Marginal distributions of need probability and consequence for the non-parametric example

Table 0.9 presents the statistics for the need probability and consequence values. The need probability was extremely low, which was not surprising considering it was the disjunction of multiple probabilities, some of which were near zero (the appropriate treatment of such cases is discussed later). The consequence was on average a positive value of \$0.56 per tonne, indicating that for the small need probability a minor benefit arose. Considering the example was, for the most part, contrived, meaningful observations were impossible to draw. Estimated confidence intervals on the mean were obtained using the percentile rank. The large uncertainty bounds for the consequence function indicated that there was a great deal of uncertainty in where the mean truly lay. Increasing the simulation size could tighten the bounds, but the purpose of the example was only to illustrate the process.

Table 0.9. Summary statistics for the need probability and consequence values

Output	Mean	Standard Deviation	Confidence interval on mean – Lower bound*	Confidence interval on mean – Upper bound*
Need probability	4.1133×10^{-5}	1.2278×10^{-4}	2.5930×10^{-8}	3.1122×10^{-4}
Consequence	0.5637	0.4950	-0.4068	1.5634
* 5% Level of significance				

7.2.8 Create a Joint Probability Density Function ($f_{P,C}(p,c)$) of Need Probability and Consequence Values

As previously mentioned, the construction of the JPDF was altered to make the process more conducive to practitioners. For the proposed methodology, no distribution building was required. If a specific value of a joint PDF was required, the only calculation would be to count all points meeting the condition and divide by the total number of simulation cycles. As an example, there were 74,660 data points where the need probability was less than 0.00006 and the consequence value exceeded zero. Thus, the estimate of the joint probability of the aforementioned conditions was 0.7466. A confidence bound on this estimate could be achieved through either: 1) running the entire simulation multiple times and recalculating the result each time, estimating the bounds from the resulting distribution or 2) using bootstrapping to resample with replacements and building a distribution of estimates of the joint distribution. Using the bootstrapping method with 5000 resamples of 10,000 data points each to estimate the 95% confidence bounds yielded the range of [0.7380, 0.7549].

7.2.9 Perform Post-Simulation Analysis as Necessary

A great deal of post-simulation analysis was possible using the proposed methodology, including benefit-cost analysis, trade-offs, sensitivity analysis, directional cosines and full lifecycle analysis. The analysis was limited to a means to compare alternatives based on the output distributions for the non-parametric example.

The steel manufacturing process was assumed to be compared to a set target for sustainability. If only the need probability was considered, then all that would be necessary was a target need probability distribution or a target average need probability. The output from the simulation in terms of need probability could then be compared directly with the target. Assuming the desired need probability was 0.000055, the current process would have a 0.1654 probability of meeting or exceeding that goal. Whether 0.1654 was an acceptable probability or not is a matter that would need to be addressed internally by comparing it to other alternatives or by using an entity designated to make such determinations.

While focusing on need probability was logical, the economics must also come be considered to account for the business side of sustainability decision making. It was assumed that the manufacturer was willing to accept at most a negative consequence of -$0.1 per ton steel, provided that it achieved a need probability equal to the mean need probability of the marginal (4.1133×10^{-5}). In this case, the probability of the joint distribution exceeding -$0.1 and 4.1133×10^{-5} was 0.1810. Once again, the interpretation of this value must be relative to a target or alternative.

Alternatively, conditional probabilities could be examined for the comparison. If the steel manufacturer had a target need probability of at least 0.00005 and at most 0.00006 and wanted to know the average consequence associated with that sustainability level, all points in the desired need probability range would need to be identified to define the conditioned set. The average consequence of those points would then need to be found. The probability of exceeding a certain consequence could also be calculated by counting the number of points meeting the set criterion and dividing by the size of the conditioned set. These values could once again serve as a comparison in order to select or reject an alternative process.

7.2.10 Define the Joint Probability Density Function for Multiple Items

The proposed alteration would still be viable in the event that multiple products were to be considered simultaneously. Generating the data to calculate the joint probability would follow the same procedure as simulating for a single product. By incorporating dependencies that exist between products (e.g. using the same or similar factories or sharing a common supplier), during the simulation phase, the non-parametric method could be used without the need to apply any dependence post-simulation. Conditionals could still be calculated as well, including those of need probabilities between products or sums of consequences. Due to the similarities between the joint distributions for multiple products and for a single product, no example is provided herein.

Whether the joint distributions for multiple products provide an accurate result is dependent upon the number of products, the number of simulation cycles, the amount of intermediate results stored and how much memory is available. Consider the case where all sustainability pillar sub-results were kept for two products using 1,000,000 simulation cycles. The total number of results that would need to be saved is 4,000,000 data pairs, or 8,000,000 individual numbers in total. Most current machines can store such files; however, some programs may not be able to keep these values in memory at the same time. There are means to address such issues. Regardless, the methodology would not be impacted.

7.2.11 Use a Spider Diagram to Compare Alternatives

As noted previously, the primary purpose of the proposed methodology was to facilitate comparisons between alternatives or an alternative against a set target. A standard way of performing such a comparison is to use a spider diagram, where an alternative's performance is plotted against another or against a target performance level. When dealing with deterministic values, this procedure is simple and effective. When the results are probability distributions however, spider diagrams can become difficult to effectively use. In this case, determining the dominance of one alternative over another or the performance relative to a desired target can be done using a sampling procedure. A point is sampled from the alternative and another is sampled from the competing alternative's distribution or some target (point estimate or distribution). The points are compared and, if the alternative is superior in terms of the criteria of interest, i.e. need probability, consequence value, or both, it is added to a running sum. This sum is divided by the total number of sampled points to determine the probability that the alternative is superior to another alternative or a target performance.

Assume the target need probability distribution for the steel manufacturing process presented in the example was a lognormal distribution with a log-mean of -10.1269 and a log-standard deviation of 0.02500. As with the other distributions used in this process, the target distribution would need to be defined either through expert elicitation or empirically. Using the sampling technique with 5000 individual draws, the probability that the steel manufacturing process exceeded the target performance was 0.21.

7.3 Non-parametric Model Example Discussion

The alteration to the DSS methodology was to increase suitability for practitioner use. By removing the reliance on DSS and moving to more traditional non-parametric techniques, the result was far more amenable to interpretation. The full range of calculations available within the original methodology are still applicable to the results after the alteration, though the interpretation changed. The DSS methodology provided a range that the true value was guaranteed to be in. By altering the procedure to easily produce a single value with confidence bounds, the ranges involved were no longer certain to contain the true result (given the input). A high degree of confidence was obtainable, but at the cost of added calculation time and the potential for high memory storage requirements.

Both methodologies provided the same type of result, albeit in different ways. The original methodology focused on providing a bounded result with greater assurance, but was less practical, while the altered methodology provided a more convenient method, but with less certainty that the true distribution has been captured. In both cases, the greatest issues were the lack of data available up front and the reliance on expert opinion elicitation to fill any gaps. Other issues, such as calculation time and memory storage for large simulations, existed for both but were more prevalent for the altered methodology. A method that relied on empirical data to determine as much of the probabilistic nature of the variables as possible would be ideal, but the effort required would be substantial. Organizational inertia against such data collection could prove difficult to overcome. Data-driven methods would be far more robust provided a sufficiently large, up-to-date and consistent database could be generated.

In general, both methods achieved the desired aim, although not without issues. The ranges of the Dempster-Schafer results and the large storage required by the non-parametric method created computational difficulties. The calculation issues for the Dempster-Shafer structures were built into the methodology itself, while the non-parametric method was an artificial limit imposed by technological capability. With greater computing power, the non-parametric methodology may become more feasible, while the Dempster-Shafer structure methodology will always rely on ranges under the assumptions of this research. Considering this, the non-parametric methodology would be preferred.

In addition to the calculation issues, the heavy reliance on expert elicitation makes the results of the methods only as accurate as the expert opinions and the data collection. Issues with expert opinion elicitation are solvable by generating the required data, even though the logistics for such a large effort preclude any immediate resolution. The potential for pushback from industries is also very high, given the nature of sustainability efforts and the regulations that may be imposed on industry.

7.4 A Comparison of Dependency Estimation Methods

Table 0.10 compares dependency estimation methods introduced in preceding sections.

Table 0.10. Comparison of Dependency Estimation Methods Mentioned in this Paper

Method	Dependency Estimation Method
Copulas	A functional form of dependency is assumed and applied using formalistic calculations.
Dempster-Shafer Structures	A generic form of dependency is assumed (unknown positive, unknown negative or unknown) is assumed and the CDF is bounded based on the widest possible bounds under those assumptions.
Nonparametric	Dependency is measured by maintaining coupling of the output pair from each simulation. By maintaining the relationship of these pairs, the entire simulation data set inherently accounts for dependency when using the appropriate table look up functions when calculating probabilities from the simulated data set.

Any of the methods in Table 0.10 could be used in the methodology without affecting accuracy of the results based on the preference or need of a practitioner; however the remainder of the work herein uses the nonparametric method for the following reasons:

- The nonparametric method makes no assumptions on the nature of the dependency in the simulated data set, instead relying on the inherent dependency that is developed by maintaining the relationship between simulation data pairs. Copulas require an assumed functional form of dependency, and DSS requires an assumed, though unknown, form.

- The results are intuitive, unlike DSS which outputs results as bounds and, in some cases, bounds of bounds.
- Simulation theoretically allows for as many data pairs as necessary to be generated, allowing for some control of the accuracy by increasing simulation cycles, if needed.

Chapter 8: Validation of Methodology

The validation of the methodology is a non-trivial task. The data required, the calculations and the use of expert elicitation makes such a process difficult. Even with appropriate data collection, the highly complex nature of the problem at hand makes finding an example where there is a "true" comparison of items that is verified and validated quite difficult. The use of pre-built databases can alleviate these issues, but it comes with the risk of not including every desired aspect of sustainability. Such databases provide the only way to generate comparisons that are validated with a consistent methodology.

The validation process utilizes NIST's Building Industry Reporting and Design for Sustainability (BIRDS) database. BIRDS is an online tool designed to evaluate the sustainability performance of commercial and residential buildings using a combination of lifecycle inventory analysis (LCIA), lifecycle cost analysis (LCC) and whole building simulation (Domich et al 2015). There are currently two separate BIRDS databases: the "Building Energy Standards/Codes Database" combining commercial and residential designs and the "Incremental Energy Efficiency Residential Building Database" for low-energy (Low-E database) buildings. The Low-E database was used in the validation process.

8.1 The BIRDS database

The Low-E database is based on NIST's Net-Zero Residential Energy Test Facility (NZERTF) located on the Gaithersburg, MD campus. The NZERTF was designed to achieve net-zero (producing as much energy as it consumes) or better performance, while maintaining the look of a typical home in the region. To achieve this the house incorporates several energy efficiency measures (EEMs) to reduce its overall energy demand (Domich et al 2015). A 10.2 kW solar photovoltaic system was placed on the roof of the building to provide on-site electricity generation. Appliances within the house are automated to turn on and off according to a schedule, and heat generators cycle on an off to simulate occupant behavior.

Validation using the BIRDS Low-E database was beneficial for the following reasons:

1. There were enough design combinations to allow for numerous different comparisons using the proposed methodology in order to gauge its relative accuracy.
2. The data was thoroughly vetted. LCA is not always consistent from practitioner to practitioner, however the Low-E database uses a consistent methodology and fully documents the data sources, many of which are from industry, providing confidence that the results are internally consistent and verifiable.
4. The database incorporates LCC analysis, allowing for validation of the cost side of the methodology.
5. There are large differences between the most efficient and least efficient designs, allowing for limiting cases to be tested under the methodology.

Differences in how BIRDS treats values must be addressed before validation can proceed, including:

1. The lack of uncertainty characterization in the BIRDS database.

2. The BIRDS database does not break out the results into operative phases, such as the construction or operation phase.

The following sections provide a brief overview of the data and processes used in the Low-E database. For a more thorough examination of the process see Kneifel et al. (2015).

8.1.1 Whole Building Simulation

The Low-E database uses the NZERTF as the basis for a whole-building energy simulation. A full factorial analysis is performed by simulating the NZERTF, while swapping out EEMs for alternative subsystems[12]. For instance, the 100% high efficiency LED lights may be replaced in the simulation by a 75% LED and 25% incandescent distribution of light fixtures. The full factorial nature of the analysis means that there are 960,000 different EEM combinations. These simulations include the following number of options for each EEM (Kneifel et al. 2015):

Windows – 5

Lighting – 4

HVAC/Ventilation/Infiltration Combination – 12 (6 gas-driven and 6 all-electric)

Domestic Hot Water (DHW) – 8 (4 gas-driven and 4 all-electric)

Wall Framing and Insulation – 5

Roof Insulation – 5

Foundation wall insulation – 4

Basement Floor Insulation – 2

[12] Subsystems considered in the analysis were: 1) windows, 2) lighting, 3) the HVAC system, 4) the ventilation system, 5) infiltration, 6) the domestic hot water system, 7) the wall framing and insulation, 8) the roof insulation, 9) the basement wall insulation, 10) the basement floor insulation, 11) Solar PV and 12) Siding

Siding – 2 (wood or brick veneer)

This paper utilized only the all-electric EEMs for the HVAC/Ventilation/Infiltration and the DHW, and only the wood siding option.

Simulations to generate the data in BIRDS were run using EnergyPlus (E+), incorporating typical meteorological year three data for the region, a three-dimensional model of the house and the occupant schedule data to obtain the energy used for each design (Drury et al. 2000).

8.1.2 Life Cycle Costing

The whole-building simulation output fed into a code that performed both LCC and LCA calculations. Calculations for the LCC component were conducted according to the American Society of Testing Materials Standard and accounted for all "cradle-to-grave" costs[13] (ASTM International). The planning horizon was 40 years maximum, with all intermediate years, staring at year one, having a separate output. Values were discounted back to present year dollars using standard TVM calculations at either a 3% or 8% discount rate. The construction was either average or luxury quality, representing additional finishes on the building during the construction process. Either 100% cash down or a 20% down 30-year mortgage was used for the initial cost of construction. Maintenance, repair and replacement were also included in the LCC calculation.

BIRDS relied on a large amount of data to develop its database. Data for the LCC are derived from multiple sources, including but not limited to:

[13] "Cradle-to-Grave" costs were: down payment, loan payments, maintenance, replacement, repair, energy costs, residual value (prorated based on remaining lifetime).

- Construction costs from RSMeans (RS Means 2015), Faithful and Gould (Faithful+Gould 2011, Faithful+Gould 2012), the US Census Bureau (2011) and local contractor quotes.
- Energy costs from Pepco's "Residential" rate schedule for Montgomery County, MD (PEPCO 2015) and US Energy Information Administration data (U.S. Energy Information Administration).
- Energy price escalation[14] rates from Lavappa et al (2017).
- MRR costs from EnergyStar (Energy Star 2011) and the National Association of Home Builders Research Center (NAHB 2007), as well as Faithful and Gould (Faithful+Gould 2011, Faithful+Gould 2012) and US Census Bureau (2011).
- Ecoinvent (2017) and academic literature for environmental data

8.1.3 Life Cycle Assessment

As with all comprehensive LCIA, the BIRDS analysis was cradle-to-grave and was grouped into different impact categories for environmental flows. All flows were tracked from raw material extraction through to final disposal, either through landfill or recycling. BIRDS used the following impact categories (units in parenthesis) (Kneifel et al. 2015):

Primary energy consumption (kBTU)

Global climate change potential (kg CO_2e)

Human health – air pollutants (kg PM_{10} eq)

Human health – cancer effects (CTUh)

[14] Estimated of the percentage change in energy prices in future years.

Human health – non-cancer effects (CTUh)

Water consumption (kg)

Ecological toxicity (CTUe)

Eutrophication potential (kg N eq)

Land use (acre)

Smog formation (kg O_3 eq)

Acidification potential (mol H+ eq)

Ozone depletion (kg CFC-11 eq)

Lifecycle assessment in BIRDS is based on a hybrid LCIA approach. The two approaches consisted of a top-down method utilizing inventory data from the Bureau of Economic Analysis (BEA) Input-Output (I-O) data (U.S. Bureau of Economic Analysis), and a bottom-up method using available inventory data from representative technologies. The bottom-up data included both primary (directly from manufacturers or participating organizations) and secondary data (from literature), as well as data collected by third party entities (U.S. LCI database and Ecoinvent database).

The top-down data used environmentally-extended I-O tables to account for the environmental flows from raw material extraction to the physical construction of a building and its maintenance. This allows the generic flows for housing construction and MRR to be determined without the need for a bottom-up analysis of every element in a house. Since the BEA I-O tables are in economic terms, the top-down approach allows for easy integration of the economics of construction.

Unlike the top-down data, the bottom-up data was specific to an EEM. For instance, a specific type of HVAC system had its own LCIA analysis. Some of the bottom-up data were derived from specific product manufacturers, while some were based on industry average values, the specifications of the EEM or published literature of tear-downs of similar products.

The bottom-up and top-down data were combined to generate the total flows of a building design over the chosen study period. BIRDS simplifies the disparate environmental flows into a single value in a different way than the proposed methodology. BIRDS normalizes the individual environmental flows by multiplying the value by the US population, and then dividing by an estimate of the total flows produced by the US for that flow. The result is a dimensionless number that represents the fractional impact of the specific flow relative to the entire nation. All the normalized flows were then weighted[15] and summed to produce an environmental impact score (EIS) that is used to compare against alternatives. While this method does solve the issue of disparate units, it does not go as far as the proposed methodology, which is:

1. Probabilistic in nature
2. Relates outcomes to sustainability goals
3. Stores input based on indicators, pillars, and lifecycle phases

The proposed methodology was adapted to work with the BIRDS output in order to validate the calculations.

8.2 Validation

The validation process did not follow all the steps in the methodology, as the data collection had already been performed (some of which was not publicly available) and

[15] Weighting options in BIRDS are: 1) BEES stakeholder panel, 2) equal weighting, 3) carbon only, 4) EPA weighting and 5) custom weighting.

adding uncertainty to the underlying calculations in BIRDS would require a near complete overhaul of the generating code. Furthermore, obtaining accurate estimates of uncertainty for the collected data was infeasible. Instead, the validation assumes that the output LCC and the intermediate flows of the LCA were the result of the proposed methodology for a single phase (entire lifecycle), two pillar (social and environmental) analysis. Indicators were taken as the environmental flows reported by BIRDS.

Conditional distributions were foregone in the analysis out of necessity as well. With the BIRDS output being deterministic, there was no way to properly estimate the dependence structure between each design's economic and sustainability performance. It was theoretically possible to build up a dependence structure, or at least a functional relationship for each EEM's sustainability performance to economic cost. But in using the totality of the designs in BIRDS, the underlying data required to build such structures was not publicly available.

It is important to note that while the term *validation* is used in this document, the validation herein is not a true validation. A true validation is a comparison between either two methods using similar methodologies, or a method against known observations. The observations in BIRDS are not true observations; they are the result of a consistent LCA and LCC methodology applied to a simulated building. While this somewhat places limitations on drawing conclusions, the BIRDS database is arguably the closest that any available dataset is going to be in terms of replicating a large amount of real world observations. The lack of an explicit use of uncertainty in BIRDS highlights the shortcoming in sustainability quantification at present.

8.2.1 Assumptions

Validation required adding uncertainty to the deterministic output of BIRDS. Uncertainty was incorporated by assuming that all BIRDS flow outputs were uniformly distributed with the bounds defined as a set multiple, referred to as the *multiplier*, away from the output value, which was the same for all distributions. If the proposed methodology worked as intended, then as the uncertainty was removed, i.e. the multiplier approached zero and the distribution approached a deterministic value defined by the BIRDS output for the flow. The results would then converge towards the BIRDS preferences, with a few caveats.

BIRDS uses a weighted summation to determine preference, while the proposed methodology uses the multiplication of probabilities and simulation. In the limiting cases this difference should have minimal or no impact on preference, however in cases where the BIRDS preference between two designs is marginal, the addition of uncertainty and the differing nature of the methods will most likely lead to preference switching. This difference can create the following scenario in Table 0.1[16]:

[16] This scenario is entirely manufactured, but does occur in the proposed methodology to BIRDS comparison.

Table 0.1. Example of preference switching condition

Indicator Design - Method	Indicator 1	Indicator 2	Indicator 3	Final EIS/NP
One – EIS	20	2	0.4	22.4
One – NP	0.8	0.75	0.01	0.006
Two – EIS	12	0.5	2	14.5
Two – NP	0.6	0.3	0.2	0.036

Based on the EISs in Table 8.1, Design One was preferred. Under the assumed distributions however, Design Two was preferred. The preference switched because multiplying a high probability by a low probability had a greater impact on the need probability (NP) results than adding a large flow and a small flow had on the EIS results. In the former situation, the low probability greatly reduced the overall NP, as a near-zero factor will bring the product close to zero (dependent on orders of magnitude). In the EIS addition, the resultant's order of magnitude was at least as large as the order of magnitude of the largest term. This situation also highlights the importance of the Analysis Specification steps in the proposed methodology, as a single indicator can have a large influence on the final measure. Whether this influence was fully considered depends on the scenario. In a case where an objective view of performance is desired then all indicators should be included even if one has an outsized impact. In a decision-making case, a single indicator having too much weight may make selecting an alternative impossible due to the precision limits the machine running the analysis. Under those conditions it may be best to omit said indicator from the formal analysis and just concede as fact that the indicator will not be met[17].

The condition created in Table 0.1 was an unavoidable result of the validation methodology and, were this strictly an example, the different results would not be an issue, as the BIRDS and the proposed methodology are fundamentally different. In the validation method, this presents a problem, as ideally the results should converge towards, if not reach, the same preference. By adding the individual indicators for each pillar together and adding uncertainty to the partial sums, the problem was somewhat mitigated as less uncertainty was artificially added to the BIRDS results. The 12 independent flows were reduced to the following two flows, based on the sustainability pillars:

- Environmental: primary energy consumption, global climate change potential, water consumption, ecological toxicity, eutrophication potential, acidification potential
- Social: human health – cancer effects, human health – non-cancer effects, human health – air pollutants, smog formation, ozone depletion

BIRDS treated these flows as independent, so the proposed methodology will as well. The analysis relied on the normalized flows from BIRDS and the *equal weighting* option.

[17] If an indicator is conceded, it is vital that it be properly reported in any final output to ensure the full picture of sustainability performance is preserved.

Target distributions were defined by a uniform distribution with a mean value equal to the average indicator level for the sum of the environmental flows for the environmental target distribution and the sum of the social flows for the target social distribution. Thus, the implicit goal of any design was to be better than the average of all the designs. The uniform distribution was selected, as it reasonably resembled the actual distribution of values in BIRDS for most indicators. Bounds for the distribution were taken as +/- 15 times the standard deviation in both cases. The multiplier had no theoretical importance, and was selected to ensure that, when drawing random flows for the simulations, the resulting need probabilities would not be zero. While not a problem in the proposed methodology, an abundance of zero need probabilities made determining preference difficult if a large number of need probabilities were the same.

The initial construction cost (IC) served as the baseline cost for all designs, as it incorporated the initial installation costs of all EEMs. The LCC served as the consequence valuation. Both had uncertainty incorporated in the same manner as the environmental flows. Due to the aforementioned lack of ability to determine dependence structures between economic and sustainable performance, the economic outputs and sustainable outputs were treated as independent.

To keep the analysis consistent, all designs used wood siding, a 3% discount rate, average construction quality and 100% cash down financing options in BIRDS.

8.2.2 Validation Methods

Two methods of validation were used:

1. Multiple random pairwise comparisons of designs

2. A ranking of 100 designs

8.2.2.1 Method 1: Pairwise Comparisons

The first method randomly selects two designs from BIRDS to compare sustainability performance in terms of EIS and the proposed non-parametric methodology. Each pairwise comparison was checked against the EIS score and evaluated assuming the EIS comparison was correct. A confusion table was generated to assess the accuracy of the comparison. The multiplier was lowered, and the proposed methodology was repeated for the same set of pairwise comparisons. At higher values of the multiplier, the uncertainty should result in more disagreement in the confusion table, while reducing the uncertainty should lead to an increase in accuracy.

Consequence valuation proceeded based on the BCR calculation in Eq. 8-1:

$$BCR = \frac{LCC_i - LCC_{AD}}{IC_i - IC_{AD}} \tag{8-1}$$

$$LCC_i \text{ is the lifecycle cost for the } i^{th} \text{ design}$$

$$LCC_{AD} \text{ is the lifecycle cost for the average design}$$

$$IC_i \text{ is the initial cost for the } i^{th} \text{ design}$$

$$IC_{AD} \text{ is the initial cost for the average design}$$

The BCR calculation was not exactly analogous to the previous definition of consequence, but it is commonly used in practical applications. Furthermore, getting consequences from the BIRDS data was infeasible.

A nearest neighbor approach was used to determine the average BCR design. The design with the LCC and initial construction cost closest to the averages determined for each under a nearest neighbor calculation was chosen. Using the average design was necessary, as the IC and LCC were not independent. A preference based on BCR was then found using both the BIRDS output and the proposed methodology.

Once the BCR was determined, the increase in need probability relative to the BCR was found in Eq. 8-2:

$$SE_{Sim} = \frac{NP}{BCR_{Sim}} \tag{8-2}$$

SE_{Sim} *is the sustainability efficiency based on the proposed methodology output*

NP *is the average need probability of the proposed methodology output*

BCR_{Sim} *is the benefit − cost ratio from the proposed methodology output*

The sustainability efficiency is a rough measure of how effective an individual design is at improving sustainability performance for its increase in BCR. The design with the highest SE_{Sim} is the preferred option. This is then compared to the ranking based on SE_{BIRDS} (as defined in Eq. 8-3):

$$SE_{BIRDS} = \frac{EIS}{BCR_{BIRDS}} \tag{8-3}$$

SE_{BIRDS} *is the sustainability efficiency based on the BIRDS output*

$$EIS \text{ is the environmental impact score using the BIRDS output}$$

$$BCR_{BIRDS} \text{ is the benefit} - \text{cost ratio using the BIRDS output}$$

Sustainable efficiency was used in lieu of the conditional distribution analysis outlined in the proposed methodology, due to the required assumption of independence between the economic output and the sustainable output. With no basis for dependence, the two output distributions were essentially independent, save for any statistically insignificant dependence that falls outside of the simulation procedure. While it would be possible to use the product of the SIR and NP (EIS for BIRDS output) distributions to estimate the probability of simultaneously achieving greater than average economic and sustainable performance, BIRDS is not set up to rank combined performance the same way. Thus the additional multiplication of probability adds more uncertainty to its deterministic process and risks a Table 0.1 condition. As such, validation requires a less probabilistic method of measuring combined performance than the proposed methodology can achieve.

8.2.2.2 Method 2: Rank Assignment

Method 2 randomly draws 100 designs and ranks the NP, BCR and SE based on BIRDS and the proposed methodology. These rankings are then compared to a ranking based on the deterministic EISs for the designs and all are plotted against a 45° line, or a line of perfect agreement. If most of the points lie on or near the line of perfect agreement, then the proposed methodology is producing results accurate to the BIRDS comparisons. As with method 1, the multiplier was lowered incrementally to check for convergence to the EIS ranking. Rankings including consequence valuation were done using BCR and the combination of consequence and environmental performance was done using SE_{Sim} comparisons to SE_{BIRDS}.

8.2.3 Validation Results

The results of the validation process presented here are for one set of randomly generated designs using a constant seed. Other seeds may produce different results. In all comparisons, 1200 simulations were examined, and a simple plurality was used to determine preference from: 1. Design One, 2. Design Two and 3. No preference. Simulations were run using a Python script.

8.2.3.1 Validation Results: Pairwise Comparisons

The confusion tables (Table 0.2) present the results of the pairwise comparisons of NP and EIS in order of descending coefficient of variation (COV) value. The multipliers examined were: 0.0001, 0.01, 1, 2, 4 and 8. In a confusion table, having most of the entries in the main diagonal meant that the two methods agreed, while off diagonal entries indicated erroneous classifications. As seen in Table 0.2, as the multiplier decreased, the number of entries in the main diagonal increased. Table 0.2-f shows a high degree of error indicating that with a multiplier of eight, the comparison resulted in an essentially random result. As the multiplier gradually decreased, the amount of erroneous entries decreased until, with a multiplier of 0.0001, there was almost perfect agreement. This suggests that, despite the differences in how the methods work, there was agreement in most pairwise comparisons. The table for 0.0001 had a Cohen's Kappa of 0.9 with a 95 % confidence bound of [0.815, 0.985], indicating strong agreement between the two methods.

Table 0.2. Confusion matrices for need probability-EIS comparison a) 0.0001 multiplier, b) 0.01 multiplier, c) 1 multiplier, d) 2 multiplier, e) 4 multiplier, f) 8 multiplier

a)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	49	4	0
	Design Two	1	46	0
	No Preference	0	0	0

b)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	49	3	0
	Design Two	1	46	0
	No Preference	0	1	0

c)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	49	4	0
	Design Two	1	43	0
	No Preference	0	3	0

d)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	43	10	0
	Design Two	7	34	0
	No Preference	0	6	0

e)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	33	19	0
	Design Two	15	30	0
	No Preference	2	1	0

f)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	27	22	0
	Design Two	22	24	0
	No Preference	1	4	0

A similar comparison was conducted for the BCR (Table 0.3). While the results for the 0.0001 multiplier showed perfect agreement, the trend in the results was different than the NP-EIS comparison. The use of the multiplier in defining the uniform distributions for the output flows in BIRDS was the cause of this difference. The multiplier was not selected based on the actual nature of the input distribution and was instead a variable used to add uncertainty to what is, in BIRDS, a deterministic value. As such, it ignored whatever unknown distribution defined the output for a particular design, bracketing the range of uncertain outputs based on multiples of the mean. If the mean was extremely large, as was the case for the LCC variable, then a plus-or-minus one mean range would may well exceed the realistic range of potential outputs by a large amount. A small value with a plus-or-minus one mean range may not deviate as much in terms of magnitude over the realistic range of variables. Consider a range of 1,000,000 plus or minus 1,000,000 compared to a range of 0.1 plus or minus 0.1. Both extend the range by a factor of two, but the potential difference in outcome relative to the mean is far larger. If smaller increments were taken for the multiplier between 1 and 0.01, a more gradual trend would be observed. The table for 0.0001 had a Cohen's Kappa of 1.0, indicating perfect agreement (Cohen's Kappa is deterministic under perfect agreement).

Table 0.3. Confusion matrices for BCR comparison a) 0.0001 multiplier, b) 0.01 multiplier, c) 1 multiplier, d) 2 multiplier, e) 4 multiplier, f) 8 multiplier

a)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	52	0	0
	Design Two	0	48	0
	No Preference	0	0	0

b)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	50	2	0
	Design Two	1	45	0
	No Preference	1	1	0

c)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	25	28	0
	Design Two	25	17	0
	No Preference	2	3	0

d)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	29	23	0
	Design Two	22	19	0
	No Preference	1	6	0

e)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	29	19	0
	Design Two	18	26	0
	No Preference	5	3	0

f)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	25	18	0
	Design Two	24	29	0
	No Preference	3	1	0

Because the SE comparison utilized the results of the BCR calculation, the pairwise comparisons in Table 0.4 followed the same trend as in Table 0.3. This merely indicates that the uncertainty in the BCR calculation was the driving force in the accuracy of the pairwise comparisons for SE values, since the SE was a calculation based both on NP and BCR. The table for 0.0001 had a Cohen Kappa of 0.98 with a 95 % confidence bound of [0.94, 1.0], indicating strong agreement between the two methods.

Table 0.4. Confusion matrices for SE comparison a) 0.0001 multiplier, b) 0.01 multiplier, c) 1 multiplier, d) 2 multiplier, e) 4 multiplier, f) 8 multiplier

a)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	52	0	0
	Design Two	1	47	0
	No Preference	0	0	0

b)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	52	4	0
	Design Two	1	42	0
	No Preference	0	1	0

c)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	29	21	0
	Design Two	19	22	0
	No Preference	5	4	0

d)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	22	24	0
	Design Two	24	22	0
	No Preference	7	1	0

e)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	28	21	0
	Design Two	21	21	0
	No Preference	4	5	0

f)

		BIRDS Preference		
		Design One	Design Two	No Preference
Method Preference	Design One	27	18	0
	Design Two	25	23	0
	No Preference	1	6	0

8.2.3.2 Validation Results: Rank Assignment

While the pairwise comparisons in the first validation method revealed how accurate the proposed methodology was when comparing two results, it did not present a broad comparison against multiple options. The second validation method covered this by ranking 100 designs based on a full factorial comparison using both the BIRDS output and the proposed methodology.

Figure 0-1 displays the results of the rankings for multipliers of 4, 2, 0.01 and 0.0001. As with the pairwise comparison, the figure displays increasing accuracy as the multiplier decreases, up to 0.01. After 0.01 there was little change in the plots. The failure to converge to the line of perfect agreement was due to the full factorial nature of the comparison. Allowing for each design to be compared against all 99 other designs increased the chances of the situation described in Table 0.1. Such a condition was more likely the farther away from the limiting cases the plot was, resulting in the 0.0001 and 0.01 plots starting to move close to the line of perfect agreement. This began at the 0th design, and then it spread out around the 12th design, increasing the spread to the 50th design, and then reversing the trend until there was near perfect agreement for the last dozen designs. Increasing the number of simulation cycles or decreasing the multiplier had no effect after 0.0001.

While not perfectly in agreement, the proposed methodology did well in matching the rankings of BIRDS, given the different manner of the calculations, with an average error in ranking of 4.83 places and a maximum absolute error of 19 places for the 0.0001 multiplier. Lin’s concordance correlation coefficient for the 0.0001 plot was 0.950 with 95% confidence bounds of [0.934, 0.962], indicating strong reproducibility between the methods.

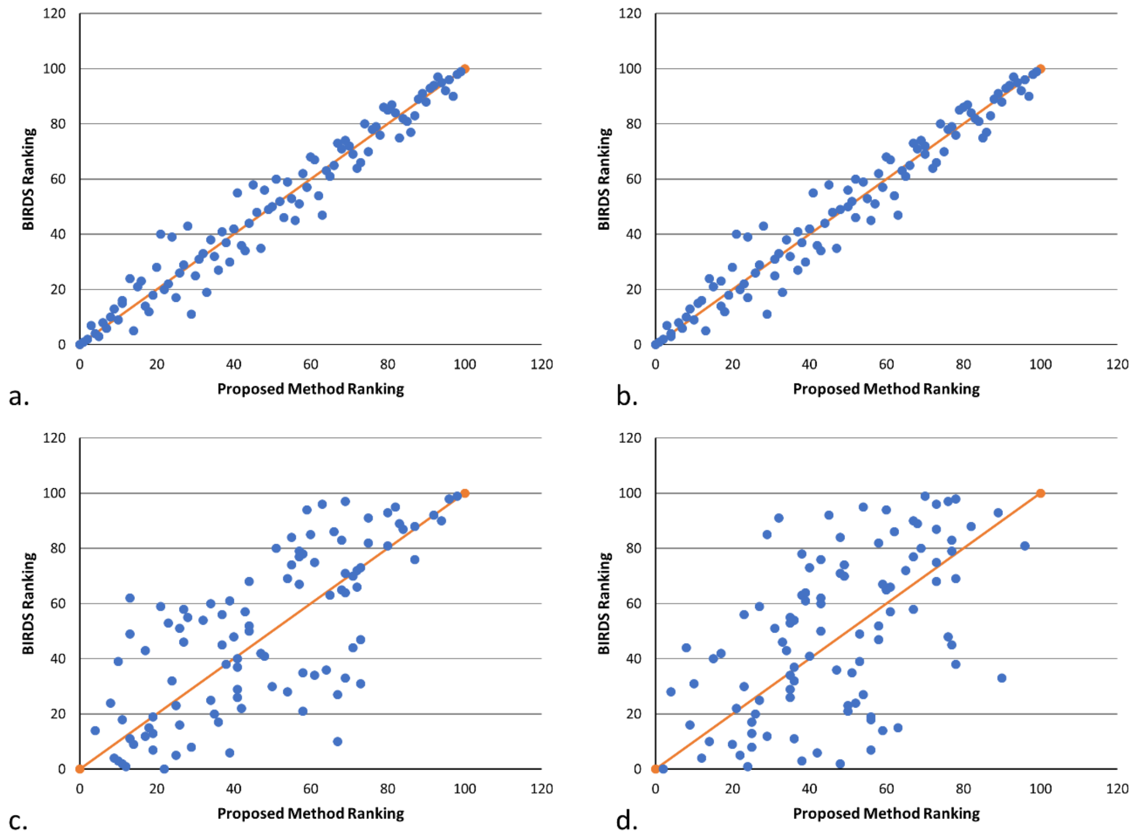


Figure 0-1. Concordance plots for NP-EIS rankings a) 0.0001 multiplier, b) 0.01 multiplier, c) 2 multiplier, d) 4 multiplier

The SIR plots in Figure 0-2, as with the SIR confusion tables, showed far greater errors for larger multipliers before converging. In the case of the SIR, the convergence was near perfect. Since the LCC and IC BIRDS outputs were treated identically in the calculation of SIR for both BIRDS and the proposed methodology, stronger convergence was expected. The average absolute error was 0.04 ranking, with an absolute maximum error of one rank. Lin's concordance correlation coefficient for the 0.0001 plot was approximately one (0.999995) with 95% confidence bounds of [0.999993, 0.999996], indicating strong reproducibility between the methods.

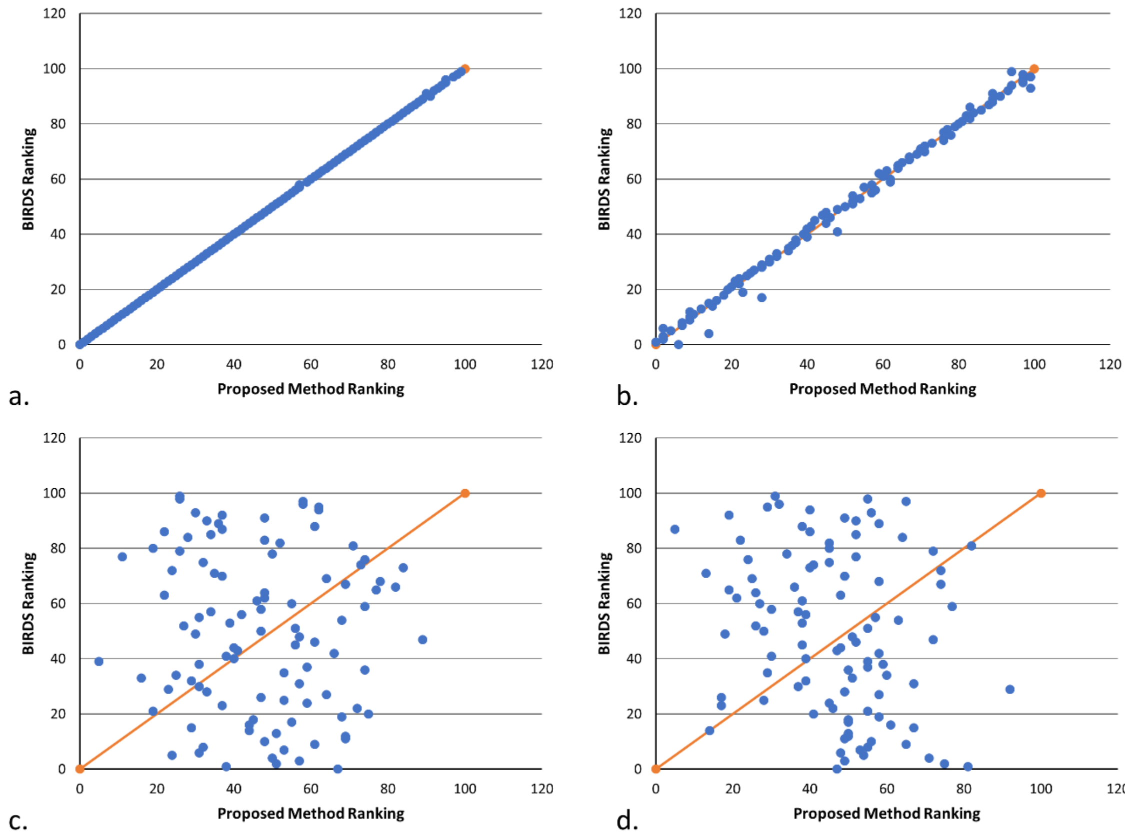


Figure 0-2. Concordance plots for BCR rankings a) 0.0001 multiplier, b) 0.01 multiplier, c) 2 multiplier, d) 4 multiplier

Examining the SE plots in Figure 0-3 revealed the same trend as in the SIR plots, except the convergence was not as strong to the line of perfect agreement. The lack of agreement in the higher multipliers was due to the combined uncertainty in the BCR and NP rankings. As the multiplier increased most of the uncertainty in the BCR was resolved, however the remaining uncertainty in the NP rankings remained. This resulted in the 0.001 showing convergence to the line of perfect agreement that was stronger than in the NP plot, however not to the same extent as in the BCR plot. The mean absolute error was 1.56 ranks with a maximum absolute error of 8 ranks. Lin's concordance correlation coefficient for the 0.0001 plot was 0.994 with 95% confidence bounds of [0.992, 0.995], indicating strong reproducibility between the methods.

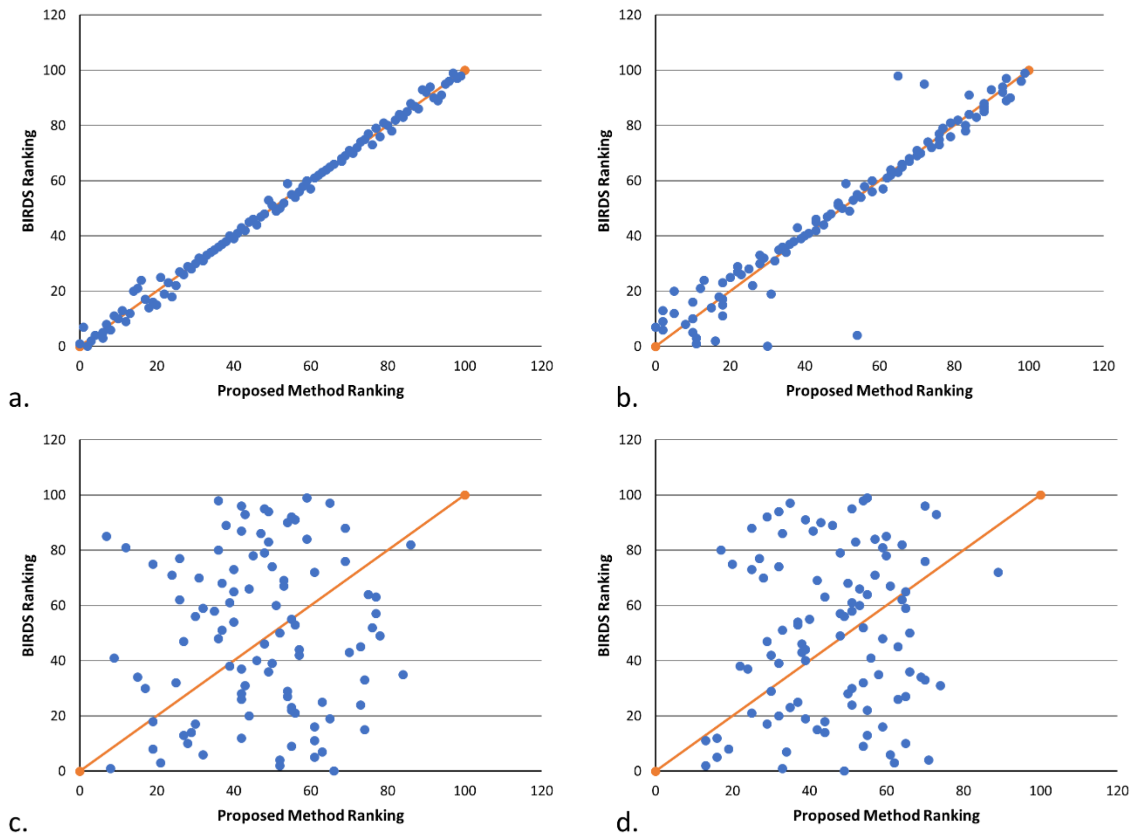


Figure 0-3. Concordance plots for SE rankings a) 0.0001 multiplier, b) 0.01 multiplier, c) 2 multiplier, d) 4 multiplier

8.3 Validation Discussion

The results of the verification methods showed that the proposed methodology could reproduce the results of a standard LCA method to a high degree of accuracy. The limiting cases in the ranked plots coupled with the strong relationship of the overall rankings with the line of perfect agreement indicated that the proposed methodology largely followed BIRDS preferences. The confusion table results showed similar agreement with a high degree of fidelity in the pairwise rankings. This all suggests that the non-parametric method is theoretically sound, and the stepwise structure ensures mathematical validity. While the differences in the methodologies make perfect agreement unlikely, the proposed methodology is close to the results from a traditional LCA approach, which gives validity to its use.

Chapter 9: Conclusion

Sustainability is an increasingly important topic as nations begin to work towards lowering their impact on the earth. While the focus of such efforts tends to be on the environmental side, awareness and consideration of the social and economic aspects of sustainability is also vital. The exploitation of vulnerable populations and the creation of any system in which people end up in untenable economic situations, due to large scale actions is a real concern and should not be overlooked. This concept is reinforced by the three pillars approach to sustainability, which is common to most definitions. To date however, difficulties in quantifying the social pillar, and a distaste for including economics in any discussion on environmental sustainability meant many methods did not include such considerations or minimized their impact.

The primary contribution of this research is a methodology that quantifies the sustainability of an item, while simultaneously incorporating uncertainty and economic considerations, which results in an output value that has intuitive meaning to practitioners and high-level decision makers regardless of experience with the method used to obtain the results. Furthermore, the methodology is validated through comparison to a thoroughly vetted and accepted data source and is flexible in its application.

The first step in developing the proposed methodology was to reinforce the ties between the three pillars in a realistic way. It acknowledged the difficult relationships between technology, science, politics, economics, society and the environment. There were limits to what was feasible at any time or place, be it political will, technological level or the degradation of the environment itself. While the ultimate goal of any sustainability effort should be the minimization of humanity's impact on it, ignoring the other realities often leads to failure. Working without consideration of the feasibility limits means pushing outside what is realistic and failing to see where work needs to be done. If there is a lack of political will to make sustainability changes, ignoring that lack of will does nothing, but recognizing it allows for work to be done in the immediate realm of what is possible. Identifying a need to increase political will through some other means is also a valuable observation. As the history of environmental degradation itself has been made clear, ignoring or pretending that a problem or boundary is not there does not make that problem go away.

The methodology was developed in order to answer the nine questions outlined in Chapter 1.1, repeated here for convenience.

1. Which indicators should be included?
2. How are the disparate units of the indicators rectified?
3. How can the methodology maintain flexibility across multiple domains?
4. How can the methodology be extended for use in other calculations?
5. Does the methodology account for all aspects of sustainability?
6. What role do economics play in the methodology?
7. How should uncertainty be considered?
8. Who are the target users, and will the methodology be useful to those users?
9. Does the methodology produce valid results?

A thorough literature review answered Question 1, and by extension Question 5, by analyzing current indicator sets and classification schemes. Indicators should be selected in a way that includes all sustainability pillars and relevant phases of a project, while omitting those that have no bearing. This means that a set group of indicators for all projects is not prescriptible, but a formulaic means of arriving at the appropriate set of indicators is feasible.

The proposed methodology answered Questions 2 and 7 by generating a means of quantifying sustainably in a way that provides meaningful interpretation beyond a comparison of alternatives and it does not ignore the uncertainty in the data that it is derived from. Instead, it seeks to establish a method that ties the impacts of a construction or manufacturing activity to the likelihood of achieving a set sustainability goal. By doing so, a probability is established that has a relatively simple meaning and can be easily understood in practice and by the general public. Stating that a design has an environmental flow of 120 CO_2e of global warming potential is a meaningful statement in an LCA, but has little meaning to someone outside of the LCA community. Alternatively, saying that a design has a 75% chance of meeting the sustainability goal of reducing its carbon footprint, as compared to an average home is something that can be communicated beyond the practitioners performing the analysis.

The methodology required the consideration of the economic impacts of sustainability efforts tied to the sustainability outcomes explicitly, which resolved Question 6. Doing so is a necessity, as sustainability will inevitably have an impact on industries economically. Incorporating economics allows for a business case to be built for those instances when sustainability efforts provide an overall benefit. But it is also useful in understanding when certain industries do not have the technological or resource capability to implement sustainability efforts, without falling into a failing business model. In an open economy, a failing business model will not survive, meaning any benefits from that model’s push for sustainability will be lost. That is not an excuse to ignore sustainability outright, but reality dictates that limitations on how much can be done at any given time must be recognized. These limitations evolve over time, due to regulations, technological breakthroughs and changes in consumer sentiment, making realistic sustainability efforts a moving target.

By using non-parametric techniques, the proposed methodology is viable for any resulting distributions that may arise from a Monte Carlo simulation. By avoiding the need to fit distributions to the output, a portion of the model uncertainty can be avoided. Although model uncertainty is not necessarily large in all cases, the large amount of uncertainty in data collection means that any steps taken to reduce the imposition of unnecessary uncertainty are advantageous. This reduction in model uncertainty comes at the cost of higher data storage needs, since every simulation must be saved, and a sufficient number must be run to allow for conditional distributions to be developed. Longer runtimes and larger output file sizes are therefore expected. This defines the user base as high-level decision makers and researchers, who are most likely to have the knowledge base to perform such calculations, resolving the first part of Question 8. Monte Carlo simulation is simple to set up and run, and the output of the methodology is simple to interpret. While it is impossible to know without the actual use of the model in the future, this at least gives a plausible reason to believe that the remainder of Question 8 may be rectified.

Questions 3 and 4 were resolved using Monte Carlo techniques and probabilities. Unlike an index value, a probability can be used with ease in other calculations. The intermediate results all maintained units allowing for analysis at intermediate levels and the only portion of the methodology tying it to a specific domain was the indicator selection step. Considering that this step is flexible enough to accept any item, provided it can fulfill the rigorous data requirements, the method can be applied to practically any domain.

A thorough validation of the methodology was performed using vetted data and output from NIST's BIRDS tool, subsequently answering Question 9. Doing so required some of the steps and processes in the proposed methodology to be omitted, since it relied on different mathematics to achieve the results. Ideally, such omissions would not be necessary, but no alternative means of validation using a thorough, well-vetted and publicly available data set existed, let alone one that incorporated both LCC and LCA calculations with uncertainty.

The validation revealed convergence to the BIRDS output, despite the addition of uncertainty and the use of a different method for calculating sustainability impacts. While not in perfect alignment, the results followed an expected trend, despite the fundamentally differing nature of the methodologies. By showing fair agreement with the BIRDS output as the uncertainty in the proposed methodology was removed, the proposed methodology revealed itself to be viable using real world data drawn from multiple resources.

The proposed methodology is not perfect. It requires a substantial amount of data collection to be utilized effectively, more so than an LCA, due to the requirement for measuring the uncertainty in inputs. A further requirement to determine the relationship between the indicator level and the likelihood of meeting a sustainability goal adds to the amount of data needed to reach a meaningful result. Much of this is work can be done by expanding the scope of traditional LCA data collection. Effort would have to made to develop widely recognized indicators for economic and societal sustainability to ensure all pillars were adequately covered. Target distributions would also have to be developed. In some cases, these distributions would be relatively simple to derive, as in, for instance, global warming potential, due to the large amount of data and study on the topic. However, others do not have the breadth of coverage, or would need to be developed locally or regionally, as not all sustainability issues appear at a global scale. Given the differences between indicators and how each may be developed, a generalized approach to defining target distributions may be infeasible from data collection to the final distribution. Many of the data issues involved with data collection for the proposed methodology exist in LCA, so the lessons learned and the advancements made in the LCA field over time may prove useful. Other analogies could be examined as well, such as partial safety factors similar to stress-strength interference and the development of probabilistic design codes.

Sustainability appears to be a topic that will be studied extensively moving forward. The need for accurate measurements in an era of concern over climate change, pollution, economic inequality and concepts of social justice will likely not abate any time soon. The proposed methodology may be considered the next step in defining the type of measures that will be needed to achieve meaningful communication of sustainability concerns and the types of data required to support the same.

Chapter 10: Future Work

While the proposed methodology lays the groundwork for a comprehensive sustainability quantification procedure, there are other issues that remain to be addressed. First there is a boundary issue. A product or building typically has an assumed lifecycle time period, however the environmental effects are not static, and have no start or end time. This is an issue that is often present in LCAs and cannot be easily addressed. The boundary for inputs and outputs is generally assumed constant through the analysis out of necessity, not necessarily representing potential changes to environment, engineering, raw material extraction, or other relevant practices. An examination in the future of how, if possible, these factors might be addressed is warranted.

The proposed methodology assumes static indicators for the life of the structure, but that is not a realistic representation of the natural world. Implementing time dependent distributions of indicators or targets to properly account for these changing effects, or altering the method to be broken into smaller discrete time steps, is required to further increase the accuracy of any analysis. Along the same lines, the discount rate in LCCs is often assumed fixed, however when discussing environmental issues it may become variable. Future generations or different stakeholders may have different ideas on how to value natural or societal impacts. Using differing, and possibly variable, discount rates, and whether or not they can be estimated, is an issue requiring further examination. The proposed methodology here may serve as a useful starting point in addressing these questions.

Appendix A Sustainability Definitions

Table A.1. General sustainability definitions

Definition	Are the Three Pillars Considered?	Applicable
"A process of change in which the exploitation of resources, the direction of investments, the orientation of technological development and institutional change are all in harmony and enhance both current and future potential to meet human needs and aspirations"	Yes	UNWECD (1987): Applicable, definition ignores financial aspects.
"The ability of systems to meet the needs of current and future generations by being physically resilient, cost-effective environmentally viable, and socially equitable."	Yes	NRC (2009): Applicable.
"A set of economic, environmental and social conditions in which all of society has the capacity and opportunity to maintain and improve its quality of life indefinitely, without degrading the quantity, quality or the availability of natural, economic and social resources."	Yes	ASCE (1996-2014): Not applicable, definition contains all pillars, however the concepts of "all of society" and "indefinitely" are difficult to actualize.
"Sustainable Development is the challenge of meeting human needs for natural resources, industrial products, energy, food, transportation, shelter, and effective waste management while conserving and protecting environmental quality and the natural resource base essential for future development."	Yes	ASCE (2009): Applicable
"The three pillars of sustainable development – economic growth, environmental stewardship, and social inclusion – carry across all sectors of development, from cities facing rapid urbanization to agriculture, infrastructure, energy development and use, water availability, and transportation."	Yes	World Bank (2014): Not applicable, the three pillars are a good starting point however it lacks some specificity for specific applications.
"The physical development and institutional operating practices that meet the needs of present users without compromising the ability of future generations to meet their own needs, particularly with regard to use and waste of natural resources. Sustainable practices support ecological, human, and economic health and vitality. Sustainability presumes that resources are finite, and should be used conservatively and wisely with a view to long-term priorities and consequences of the ways in which resources are used."	Yes	UCLA (2014): Not applicable, limits considerations to use and waste of natural resources.
"Sustainable development involves devising a social and economic system, which ensures that these goals are sustained, i.e. that real incomes rise, educational standards increase, the health of the nation improves, and the general quality of life is advanced."	Yes	Pearce et al. (1989): Applicable
"Sustainable development is concerned with the development of a society where the costs of development are not transferred to future generations, or at least an attempt is made to compensate for such costs."	No	Pearce (1993): Applicable if "costs" in the definition are expanded to three pillars

"The entropic physical flow from nature's sources through the economy and back to nature's sinks, is to be non-declining. More exactly, the capacity of the ecosystem to sustain those flows is not to be run down. Natural capital is to be kept intact."	No	Daly (2002): Not applicable, ignores social aspect of sustainability
"The utility [to] future generations is to be non-declining."	No	Daly (2002): Not applicable, no environmental or social consideration

Table A.2. General and societal sustainability definitions

Definition	Are the Three Pillars Considered?	Source: Applicable
"[To] create and maintain conditions, under which humans and nature can exist in productive harmony, that permit fulfilling the social, economic, and other requirements of present and future generations."	Yes	NEPA (1969): Applicable, drives EPA policy
"The ability to meet the needs of present without compromising the ability of future generations to meet their own needs"	No	UNWECD (1987): Not applicable in meaningful context, provided good starting point
"The level of human consumption and activity, which can continue in the foreseeable future, so that the systems that provide goods and services to the humans, persist indefinitely"	No	NRC (2009): Not applicable, ignores economic pillar
"Meeting today's economic, social, and environmental needs while enhancing the ability of future generations to meet their economic, social, and environmental needs"	yes	NRC (2009): Applicable
"A society that can persist over generations; one that is far-seeing enough, flexible enough, and wise enough not to undermine either its physical or its social system of support"	No	NRC (2013): Not applicable, does not consider economic pillar
"Creating and maintaining conditions under which humans and nature can exist in productive harmony and that permit fulfilling social, economic, and other requirements of present and future generations.'	Yes	EPA (2012) Applicable, informs government policy
"Three pillars; Environment, Society, Economy"	Yes	Multiple e.g., EPA (2013b): Not applicable, does not provide any meaningful definition
"[1] Able to be used without being completely used up or destroyed [2] Involving methods that do not completely use up or destroy natural resources [3] Able to last or continue for a long time"	No	Merriam-Webster (2014): Not applicable, basic definitions not pertaining specifically to sustainability in an engineering context

Table A.3. General sustainable manufacturing definitions

Definition	Are the Three Pillars Considered?	Source: Applicability
"The creation of manufactured products through economically-sound processes that minimize negative environmental impacts while conserving energy and natural resources. Sustainable manufacturing also protects employee, community, and consumer safety."	Yes	EPA (2013a): Applicable
"Sustainable manufacturing is defined as the creation of manufactured products that use processes that minimize negative environmental impacts, conserve energy and natural resources, are safe for employees, communities, and consumers and are economically sound."	Yes	USDOC (2009): Applicable, essentially rewording of EPA (2013a) definition.
"A systems approach for the creation and distribution (supply chain) of innovative products and services that: minimizes resources (inputs such as materials, energy, water, and land); eliminates toxic substances; and produces zero waste that in effect reduces greenhouse gases, e.g., carbon intensity, across the entire life cycle of products and services."	No	Sudarsan (2010): Not applicable, ignores economic and social aspects.
"[Reducing] the intensity of materials use, energy consumption, emissions, and the creation of unwanted by-products while maintaining, or improving, the value of products to society and to organizations."	Yes	OECD (2009): Applicable, USDOC or EPA preferred as it has government backing.
"The creation of goods or services that satisfy customer needs while respecting the environment and communities' wellbeing."	Yes	Badurdeena (2010): Applicable

Table A.4. Supply-chain sustainable manufacturing definitions

Definition	Are the Three Pillars Considered?	Source: Applicability
“The management of environmental, social and economic impacts, and the encouragement of good governance practices, throughout the lifecycles of goods and services.”	Yes	Sisco (2010): Applicable
“The process of using environmentally friendly inputs and transforming these inputs through change agents - whose byproducts can improve or be recycled within the existing environment. This process develops outputs that can be reclaimed and re-used at the end of their life-cycle thus, creating a sustainable supply chain.”	No	Penfield (2008): Not applicable, no social pillar consideration
“Must operate within a realistic financial structure, as well as contribute value to our society...must take account of all relevant economic, social, and environmental issues.”	Yes	Centinkaya (2011): Not applicable, not a full definition but provides guideline for one
“Sustainability also must integrate issues and flows that extend beyond the core of SCM: product design, manufacturing by-products, by-products produced during product use, product life extension, product end-of-life, and recovery processes at end-of-life.”	No	Linton (2007): Not applicable, no economic or social consideration
“Management of our supply base to drive affordability and innovation through social responsibility and environmental stewardship”	Yes	Lockheed (2013): Applicable

Table A.5. Sustainable construction definitions

Definition	Are the Three Pillars Considered?	Source: Applicability
"The practice of increasing the efficiency with which buildings and their sites use and harvest energy, water, and materials and protecting and restoring human health and the environment, throughout the building life-cycle: siting, design, construction, operation, maintenance, renovation and deconstruction. The practice of creating and using healthier and more resource-efficient models of construction, renovation, operation, maintenance and demolition."	No	EPA (2012): Not applicable, ignores economic sustainability
"The adoption of building designs, construction methods and materials that are environmentally friendly. It also means using materials and resources that have sustainable supplies and are readily available from many sources."	No	Tiat (2007): Not applicable, explicit consideration of suppliers is good, however neglects social and economic factors.
"Designing and constructing houses that are efficient and durable, that use less resources, are healthy to live in and are affordable."	No	Habitat for Humanity (2014): Not applicable, does not take into account full environmental, social, and economic impacts.
"The need to find a balance between economic, environmental and social factors in the design, construction and use of buildings."	Yes	University of Southampton (2008): Applicable
"Those buildings that have minimum adverse impacts on the built and natural environment, in terms of the buildings themselves, their immediate surroundings and the broader regional and global." Sustainable building" may be defined as building practices, which strive for integral quality (including economic, social and environmental performance) in a very broad way. Thus, the rational use of natural resources and appropriate management of the building stock will contribute to saving scarce resources, reducing energy consumption (energy conservation), and improving environmental quality."	Yes	Sassi (2006): Applicable
"Healthy facilities designed and built in a resource-efficient manner, using ecologically based principles."	No	Kibert (2008): Not applicable, does not consider all pillars
"A high-performance property that considers and reduces its impact on the environment and human health."	No	Yudelson (2008): Not applicable, does not consider all pillars
"Sustainable construction refers to construction activities whose negative impacts are minimized and positive impacts maximized so as to achieve a balance in terms of environmental, economic and social performance."	No	Zabihi (2012): Not applicable, does not consider all pillars

Appendix B Simulation Methodology Using Weighting

Another method for incorporating the relative importance of indicators is through an ad hoc weighting procedure. The development of weights can utilize any acceptable method, such as the analytical hierarchical process methodology, survey weighting, stratum weighting, etc. Thus, the only changes to the methodology are: (1) the removal of the adjustment of the conversion distributions in Chapter 6.1.5 and (2) the adjustment to the simulation procedure described in this Appendix.

Simulation produces the probabilities of each indicator being met individually. Finding the intersection of these and incorporating weights requires additional computation. Any computational method used must meet two conditions: (1) it must be reducible to the intersection of independent events for equally weighted values and (2) it must be consistent with the properties of weighting, that is if a value is given a higher weighting the result of the calculation should be closer to that value. A method that meets these requirements involves using the geometric average based on the assumption of independence. In the context of this procedure, the geometric average gives the average probability of arriving at a single value out of all those taken in the average calculation. The geometric average also calculates a value with the properties given below:

$$\bar{x} = \exp\left(\frac{\sum_{i=1}^{n} w_i \ln(p_i)}{\sum_{i=1}^{n} w_i}\right) \tag{B-1}$$

If the average contains three values, equally weighted, as given in equation B-2:

$$\tilde{x} = \{x_1, x_2, x_3\} \quad \text{(B-2)}$$

$$\bar{x} = \exp\left(\frac{\frac{1}{3}\ln(x_1) + \frac{1}{3}\ln(x_2) + \frac{1}{3}\ln(x_3)}{1}\right) \quad \text{(B-3)}$$

then the equation simplifies to

$$(\bar{x})^3 = x_1 x_2 x_3 \quad \text{(B-4)}$$

To determine the total probability, this value must be raised to the power of however many indicators are within the phase-pillar group. A group is defined as a collection of indicators that are within the same construction phase and sustainability pillar. Equation B-5 provides the general formula.

$$p_G = \left(\exp\left(\frac{\sum_{i=1}^{n} w_i \ln(p_i)}{\sum_{i=1}^{n} w_i}\right)\right)^n \quad \text{(B-5)}$$

$$p_G = probability\ of\ meeting\ group\ sustainability\ needs$$

$$n = number\ of\ indicators$$

$$w_i = indicator\ wieghting\ factor$$

$$p_i = probability\ of\ i^{th}\ indicator\ meeting\ sustainability\ needs$$

Equation B-6 can be further simplified to:

$$p_G = \prod_{i=1}^{n} p_i^{n(w_i)} \tag{B-6}$$

For the probability of meeting all three pillars within a lifecycle phase, a similar process can be followed:

$$p_{Phase} = \prod_{i=1}^{3} {p_{G_i}}^{3\left(w_{pillar_i}\right)} \tag{B-7}$$

$$p_{Phase} = probability\ of\ meeting\ phase\ sustainability\ needs$$

$$w_{pillar_i} = pillar\ wieghting\ factor$$

By focusing on phases, the most detrimental portion of the structure's lifecycle can be targeted for potential improvement. Final need probability follows Eq. B-8:

$$p_{Total} = \prod_{i=1}^{l} p_i^{l\left(w_{phase_i}\right)} \tag{B-8}$$

$$p_{Total} = probability\ of\ meeting\ all\ sustainability\ needs$$

$$l = number\ of\ phases$$

$$w_i^{phase} = phase\ wieghting\ factor$$

Using Eq. B-2 through B-4, each simulation cycle will produce a set of probabilities for meeting sustainability needs for each group, phase and total aggregation. Alternatively, Equations B-9 and B-10 provide aggregation within the pillars first, and then in total.

$$p_{Pillar} = \prod_{i=1}^{m} {p_{SG_i}}^{m\left(w_i^{phase}\right)} \tag{B-9}$$

$$p_{Pillar} = probability\ of\ meeting\ all\ sustainability\ needs$$

$$p_{SG} = probability\ of\ meeting\ subgroup\ sustainability\ needs$$

$$m = number\ of\ phases$$

$$w_i^{phase} = phase\ wieghting\ factor$$

$$p_{Total} = \prod_{i=1}^{3} {p_{Pillar_i}}^{3\left(w_i^{pillar}\right)} \tag{B-10}$$

$$w_i^{pillar} = pillar\ wieghting\ factor$$

Distributions can then be fit to all desired result levels. Traditional statistical analysis can follow as required. Of particular importance are sensitivity analysis, confidence bounds and directional cosines.

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